Surface Areas and Volumes

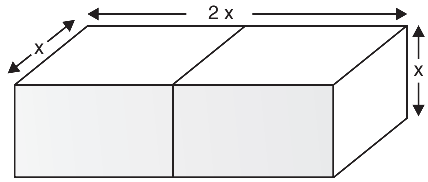
NCERT TEXTBOOK QUESTIONS SOLVED

EXERCISE 13.1

[Unless stated otherwise, take $\pi = \frac{22}{7}$*]*

Q. 1. *Two cubes each of volume 64 cm*^{3} *are joined end to end. Find the surface area of the resulting cuboid.*

Sol. Volume of each cube = 64 cm^3



$\therefore$ Total volume of the two cubes $= 2 \times 64 \text{ cm}^3$

$= 128 \text{ cm}^3$

Let the edge of each cube = x

$\therefore \quad x^3 = 64 = 4^3$

$\Rightarrow \quad x = 4 \text{ cm}$

Now, Length of the resulting cuboid $l = 2x$ cm

Breadth of the resulting cuboid $b = x$ cm

Height of the resulting cuboid $h = x$ cm

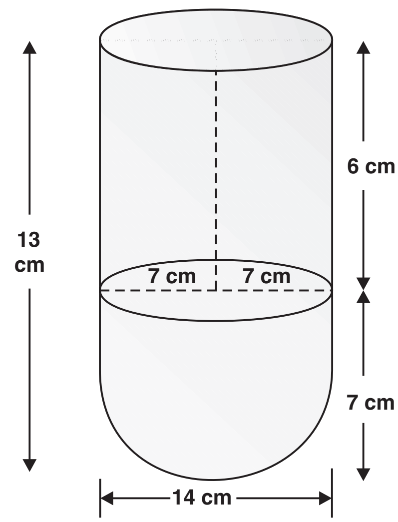
$\therefore$ Surface area of the cuboid $= 2\,(lb + bh + hl) = 2\,[(2x \cdot x) + (x \cdot x) + (x \cdot 2x)]$

$= 2\,[(2 \times 4 \times 4) + (4 \times 4) + (4 \times 2 \times 4)] \text{ cm}^2$

$= 2\,[32 + 16 + 32] \text{ cm}^2 = 2\,[80] \text{ cm}^2 =$ **160 cm²**.

Q. 2. *A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel.*

Sol. For cylindrical part:



Radius $(r) = 7$ cm

Height $(h) = 6$ cm

$\therefore$ Curved surface area

$= 2\pi rh$

$= 2 \times \dfrac{22}{7} \times 7 \times 6 \text{ cm}^2 = 264 \text{ cm}^2$

For hemispherical part:

Radius $(r) = 7$ cm

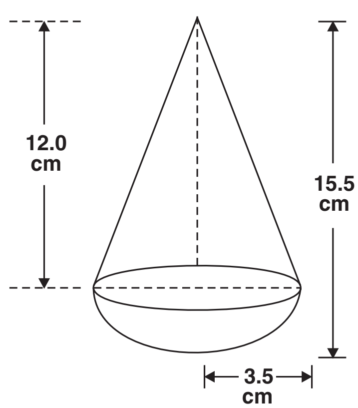
$\therefore$ Surface area $= 2\pi r^2$

$= 2 \times \dfrac{22}{7} \times 7 \times 7 \text{ cm}^2 = 308 \text{ cm}^2$

$\therefore$ Total surface area

$= (264 + 308) \text{ cm}^2 =$ **572 cm²**.

Q. 3. *A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.* [CBSE 2012]



Sol. Here, $r = 3.5$ cm

$\therefore \quad h = (15.5 - 3.5) \text{ cm} = 12.0 \text{ cm}$

Surface area of the conical part

$= \pi rl$

Surface area of the hemispherical part

$= 2\pi r^2$

∴ Total surface area of the toy

$= \pi rl + 2\pi r^2 = \pi r\,(l + 2r)$ cm^2

$= \frac{22}{7} \times \frac{35}{10}\ (12.5 + 2 \times 3.5)$ cm^2

$= 11 \times (12.5 + 7)$ cm^2

$= 11 \times 19.5$ cm^2 = **214.5 cm^2.**

$\because l^2 = (12)^2 + (3.5)^2$

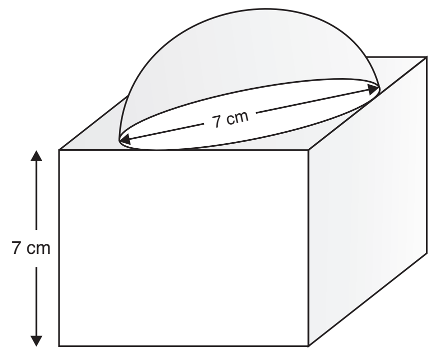
$l^2 = 144 + 12.25 = 156.25$

$\Rightarrow l = 12.5$ cm

Q. 4. *A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.*

Sol. Side of the block = 7 cm

⇒ The greatest diameter of the hemisphere = 7 cm



Surface area of the solid

= [Total S.A. of the cubical block]

+ [S.A. of the hemisphere]

– [Base area of the hemisphere]

$= (6 \times l^2) + 2\pi r^2 - \pi r^2$

[where $l = 7$ cm and $r = \frac{7}{2}$ cm]

$$= \left(6 \times 7^2\right) + \left(2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right) - \left(\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right) \text{cm}^2$$

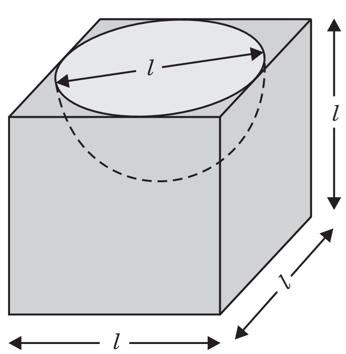
$$= (6 \times 49) + (11 \times 7) - \left(11 \times \frac{7}{2}\right) \text{cm}^2 = \left(294 + 77 - \frac{77}{2}\right) \text{cm}^2 = \mathbf{332.5\ cm^2.}$$

Q. 5. *A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid.* [AI. CBSE (Foreign) 2014]

Sol. Let 'l' be the side of the cube.

∴ The greatest diameter of the curved hemisphere = l

⇒ Radius of the curved hemisphere = $\frac{l}{2}$



∴ Surface area of hemisphere = $2\pi r^2$

$$= 2 \times \pi \times \frac{l}{2} \times \frac{l}{2} = \frac{\pi l^2}{2}$$

Base area of the hemisphere = $\pi\left(\frac{l}{2}\right)^2 = \frac{\pi l^2}{4}$

Surface area of the cube = $6 \times l^2 = 6l^2$

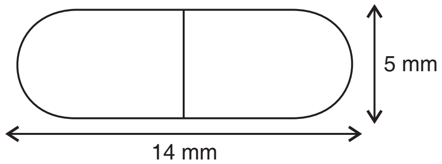
∴ Surface area of the remaining solid

$$= 6l^2 + \frac{\pi l^2}{2} - \frac{\pi l^2}{4} = \frac{24l^2 + 2\pi l^2 - \pi l^2}{4} = \frac{24l^2 + \pi l^2}{4}$$

$$= \frac{l^2}{4}\ \mathbf{(24 + \pi)\ sq.\ units.}$$

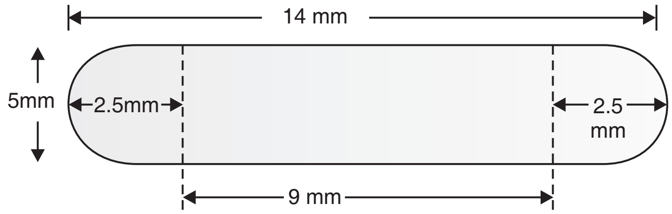
Q. 6. *A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (see Fig.). The length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.*

[CBSE 2012]



Sol. Radius of the hemispherical part

$$= \frac{5}{2}\text{ mm} = 2.5\text{ mm}$$



$\therefore$ Surface area of one hemispherical part = $2\pi r^2$

$\Rightarrow$ Surface area of both hemispherical parts

$$= 2\,(2\pi r^2) = 4\pi r^2 = 4 \times \frac{22}{7} \times \left(\frac{25}{10}\right)^2 \text{ mm}^2$$

$$= 4 \times \frac{22}{7} \times \frac{25}{10} \times \frac{25}{10} \text{ mm}^2$$

Area of cylindrical part $= 2\pi rh = 2 \times \frac{22}{7} \times 2.5 \times 9 \text{ mm}^2 = 2 \times \frac{22}{7} \times \frac{25}{10} \times 9 \text{ mm}^2$

$\therefore$ Total surface area $= \left[2 \times \frac{22}{7} \times \frac{25}{10} \times 9\right] + \left[4 \times \frac{22}{7} \times \frac{25}{10} \times \frac{25}{10}\right] \text{mm}^2$

$$= \left(2 \times \frac{22}{7} \times \frac{25}{10}\right)\left[9 + \frac{50}{10}\right] \text{mm}^2 = \frac{44 \times 25}{70} \times 14 \text{ mm}^2$$

$$= \frac{44 \times 25 \times 2}{10} \text{ mm}^2 = 44 \times 5 \text{ cm}^2 = \mathbf{220\ mm^2.}$$

Q. 7. *A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindrical part are 2.1 m and 4 m respectively, and the slant height of the top is 2.8 m, find the area of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of ₹ 500 per m*^{2}*. (Note that the base of the tent will not be covered with canvas.)*

Sol. For cylindrical part:

$$\text{Radius } (r) = \frac{4}{2} \text{ m} = 2 \text{ m}$$

$$\text{Height } (h) = 2.1 \text{ m}$$

$$\therefore \text{ Curved surface area} = 2\pi rh = 2 \times \frac{22}{7} \times 2 \times \frac{21}{10} \text{ m}^2$$

For conical part:

$$\text{Slant height } (l) = 2.8 \text{ m}$$

$$\text{Base radius } (r) = 2 \text{ m}$$

$$\therefore \quad \text{Curved surface area} = \pi rl = \frac{22}{7} \times 2 \times \frac{28}{10} \text{ m}^2$$

$\therefore$ **Total surface area**

= [Surface area of the cylindrical part] + [Surface area of conical part]

$$= \left[2 \times \frac{22}{7} \times 2 \times \frac{21}{10}\right] + \left[\frac{22}{7} \times 2 \times \frac{28}{10}\right] \text{m}^2$$

$$= 2 \times \frac{22}{7}\left[\frac{42}{10} + \frac{28}{10}\right] \text{m}^2 = 2 \times \frac{22}{7} \times \frac{70}{10} \text{ m}^2 = \mathbf{44 \text{ m}^2}$$

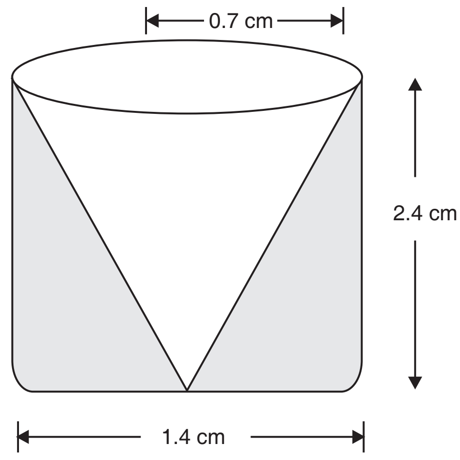
Cost of the canvas used:

Cost of 1 m^2 of canvas = ₹ 500

$\therefore$ Cost of 44 m^2 of canvas = ₹ 500 × 44 = **₹ 22000.**

Q. 8. *From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm².* [CBSE 2012, CBSE (Delhi) 2014]

Sol. For cylindrical part:



$$\text{Height} = 2.4 \text{ cm}$$

$$\text{Diameter} = 1.4 \text{ cm}$$

$$\Rightarrow \quad \text{Radius } (r) = 0.7 \text{ cm}$$

$\therefore$ Total surface area of the cylindrical part

$$= 2\pi rh + 2\pi r^2 = 2\pi r\,[h + r]$$

$$= 2 \times \frac{22}{7} \times \frac{7}{10}\,[2.4 + 0.7] \text{ cm}^2$$

$$= \frac{44}{10} \times 3.1 \text{ cm}^2 = \frac{44 \times 31}{100} = \mathbf{\frac{1364}{100} \text{ cm}^2}$$

For conical part:

$$\text{Base area } (r) = 0.7 \text{ cm}$$

$$\text{Height } (h) = 2.4 \text{ cm}$$

$$\therefore \quad \text{Slant height } (l) = \sqrt{r^2 + h^2} = \sqrt{(0.7)^2 + (2.4)^2} = \sqrt{0.49 + 5.76} = \sqrt{6.25} = 2.5 \text{ cm}$$

$\therefore$ Curved surface area of the conical part

$$= \pi rl = \frac{22}{7} \times 0.7 \times 2.5 \text{ cm}^2 = \frac{22}{7} \times \frac{7}{10} \times \frac{25}{10} \text{cm}^2 = \frac{22 \times 25}{100} \text{cm}^2 = \mathbf{\frac{550}{100} \text{ cm}^2}$$

Base area of the conical part

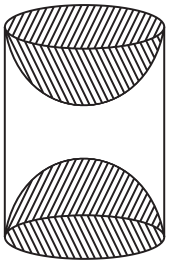
$$= \pi r^2 = \frac{22}{7} \times \left(\frac{7}{10}\right)^2 \text{cm}^2 = \frac{22 \times 7}{100} \text{cm}^2 = \mathbf{\frac{154}{100} \text{cm}^2}$$

Total surface area of the remaining solid

$$= \left[\begin{pmatrix}\text{Total SA of cylindrical} \\ \text{part}\end{pmatrix} + \begin{pmatrix}\text{Curved surface area} \\ \text{of conical part}\end{pmatrix}\right] - \begin{pmatrix}\text{Base area of the} \\ \text{conical part}\end{pmatrix}$$

$$= \left[\frac{1364}{100} \text{cm}^2 + \frac{550}{100} \text{cm}^2\right] - \frac{154}{100} \text{cm}^2 = \frac{1914}{100} \text{cm}^2 - \frac{154}{100} \text{cm}^2 = \frac{1760}{100} \text{cm}^2 = \mathbf{17.6 \text{ cm}^2}.$$

Q. 9. *A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in Fig. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article.*



Sol. Radius of the cylinder (r) = 3.5 cm

Height of the cylinder (h) = 10 cm

∴ Total surface area $= 2\pi rh + 2\pi r^2 = 2\pi r (h + r)$

$$= 2 \times \frac{22}{7} \times \frac{35}{10}\left(10 + \frac{35}{10}\right) \text{cm}^2$$

$$= 22\left(\frac{135}{10}\right) \text{cm}^2 = \mathbf{297 \text{ cm}^2}$$

Curved surface area of a hemisphere $= 2\pi r^2$

∴ Curved surface area of both hemispheres

$$= 2 \times 2\pi r^2 = 4\pi r^2 = 4 \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10} \text{cm}^2 = \mathbf{154 \text{ cm}^2}$$

Base area of a hemisphere $= \pi r^2$

∴ Base area of both hemispheres $= 2\pi r^2$

$$= 2 \times \frac{22}{7} \times (3.5)^2 = \frac{2 \times 22 \times 35 \times 35}{7 \times 100} \text{cm}^2 = \mathbf{77 \text{ cm}^2}$$

∴ Total surface area of the remaining solid

$$= 297 \text{ cm}^2 + 154 \text{ cm}^2 - 77 \text{ cm}^2 = (451 - 77) \text{ cm}^2 = \mathbf{374 \text{ cm}^2}.$$

- **Volume of a Combination of Solids**

Now, we shall calculate the volumes of combinations of two basic solids. The volume of the solid formed by joining two basic solids will actually be the sum of the volumes of the constituents.

NCERT TEXTBOOK QUESTIONS SOLVED

EXERCISE 13.2

[Unless stated otherwise, take $\pi = \frac{22}{7}$*]*

Q. 1. *A solid is in the shape of a cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid in terms of* π.

Sol. Here, $r = 1$ cm and $h = 1$ cm.

$\because$ Volume of the conical part $= \frac{1}{3}\pi r^2 h$

Volume of the hemispherical part $= \frac{2}{3}\pi r^3$

$\therefore$ Volume of the solid shape $= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2 [h + 2r]$

$= \frac{1}{3}\pi (1)^2 [1 + 2(1)] \text{ cm}^3 = \frac{1}{3}\pi \times 1 \times [3] \text{ cm}^3$

$= \frac{3\pi}{3} \text{ cm}^3 = \boldsymbol{\pi} \textbf{ cm}^3.$

Q. 2. *Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.)*

Sol. Here, diameter = 3 cm

$\Rightarrow$ Radius $(r) = \frac{3}{2}$ cm

Total height = 12 cm

Height of a cone (h_1) = 2 cm

$\therefore$ Height of both cones $= 2 \times 2 = 4$ cm

$\Rightarrow$ Height of the cylinder $(h_2) = (12 - 4)$ cm $= 8$ cm.

Now, volume of the cylindrical part $= \pi r^2 h_2$

Volume of both conical parts $= 2\left[\frac{1}{3}\pi r^2 h_1\right]$

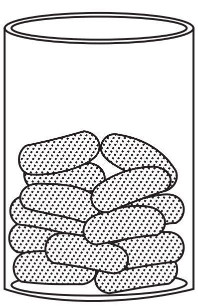
$\therefore$ Volume of the whole model

$= \pi r^2 h_2 + \frac{2}{3}\pi r^2 h_1 = \pi r^2\left[h_2 + \frac{2}{3}h_1\right]$

$= \frac{22}{7} \times \left(\frac{3}{2}\right)^2 \left[8 + \frac{2}{3}(2)\right] \text{cm}^3 = \frac{22}{7} \times \frac{9}{4} \times \left(\frac{24+4}{3}\right) \text{cm}^3$

$= \frac{22}{7} \times \frac{3}{4} \times 28 \text{ cm}^3 = 22 \times 3 \text{ cm}^3 = \textbf{66 cm}^3.$

Q. 3. *A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm (see figure).* [CBSE 2008]



Sol. Since, a gulab jamun is like a cylinder with hemispherical ends.

Total height of the gulab jamun = 5 cm.

Diameter = 2.8 cm

$\Rightarrow$ Radius = 1.4 cm

$\therefore$ Length (height) of the cylindrical part = 5 cm – (1.4 + 1.4) cm

$= 5 \text{ cm} - 2.8 \text{ cm} = 2.2 \text{ cm}$

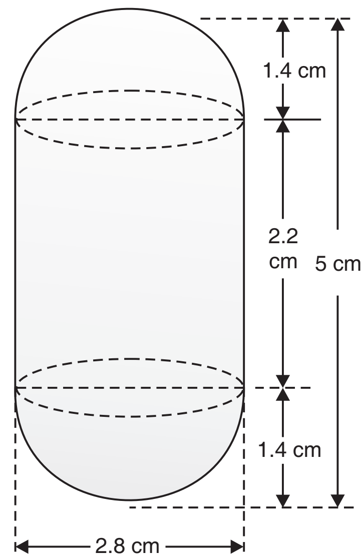
Now, volume of the cylindrical part

$= \pi r^2 h$

Volume of a hemispherical end $= \frac{2}{3}\pi r^3$

Volume of both the hemispherical ends

$= 2\left(\frac{2}{3}\pi r^3\right) = \frac{4}{3}\pi r^3$



∴ Volume of a gulab jamun

$$= \pi r^2 h + \frac{4}{3}\pi r^3$$

$$= \pi r^2\left[h + \frac{4}{3}r\right]$$

$$= \frac{22}{7} \times (1.4)^2\left[2.2 + \frac{4}{3}(1.4)\right] \text{cm}^3$$

$$= \frac{22}{7} \times \frac{14}{10} \times \frac{14}{10}\left[\frac{22}{10} + \frac{56}{30}\right] \text{cm}^3$$

$$= \frac{22 \times 2 \times 14}{10 \times 10}\left[\frac{66+56}{30}\right] \text{cm}^3 = \frac{44 \times 14}{100} \times \frac{122}{30} \text{cm}^3$$

⇒ Volume of 45 gulab jamuns

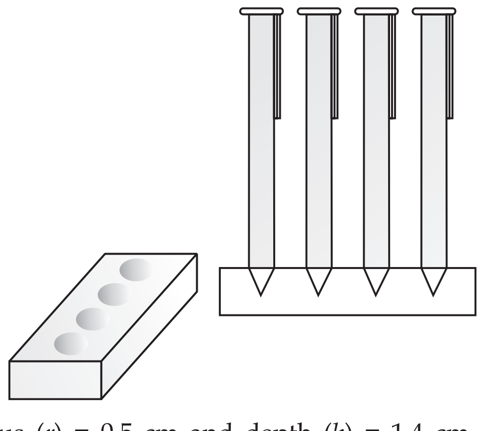
$$= 45 \times \left[\frac{44 \times 14}{100} \times \frac{122}{30}\right] \text{cm}^3 = \frac{15 \times 44 \times 14 \times 122}{1000} \text{cm}^3$$

Since, the quantity of syrup in gulab jamuns

$$= 30\% \text{ of [volume]} = 30\% \text{ of} \left[\frac{15 \times 44 \times 14 \times 122}{1000}\right] \text{cm}^3$$

$$= \frac{30}{100} \times \frac{15 \times 44 \times 14 \times 122}{1000} \text{cm}^3 = 338.184 \text{ cm}^3 = \mathbf{338 \text{ cm}^3} \text{ (approx.)}$$

Q. 4. *A pen stand made of wood is in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. Find the volume of wood in the entire stand (see Fig.).* [NCERT Exemplar]



Sol. Dimensions of the cuboid are 15 cm, 10 cm and 3.5 cm.

∴ Volume of the cuboid $= 15 \times 10 \times \frac{35}{10} \text{ cm}^3$

$= 15 \times 35 \text{ cm}^3$

$= 525 \text{ cm}^3$

Since each depression is conical with base radius (r) = 0.5 cm and depth (h) = 1.4 cm,

∴ Volume of each depression (cone)

$$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times \left(\frac{5}{10}\right)^2 \times \frac{14}{10} \text{ cm}^3$$

Since there are 4 depressions,

∴ Total volume of 4 depressions

$$= 4 \times \frac{1}{3} \times \frac{22}{7} \times \frac{5}{10} \times \frac{5}{10} \times \frac{14}{10} \text{ cm}^3 = \frac{4}{3} \times \frac{11}{10} \text{ cm}^3 = \frac{44}{30} \text{ cm}^3$$

Now, volume of the wood in entire stand

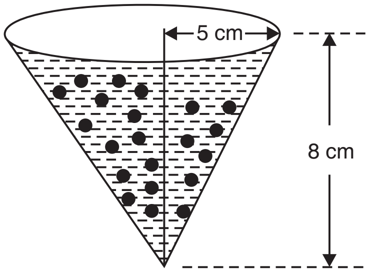
= [Volume of the wooden cuboid] – [Volume of 4 depressions]

$$= 525 \text{ cm}^3 - \frac{44}{30} \text{ cm}^3 = \frac{15750 - 44}{30} \text{ cm}^3 = \frac{15706}{30} \text{ cm}^3 = \mathbf{523.53 \text{ cm}^3}.$$

Q. 5. *A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to the brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.*

Sol. Height of the conical vessel (h) = 8 cm

Base radius (r) = 5 cm



∴ Volume of the cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times (5)^2 \times 8 \text{ cm}^3$$

$$= \frac{4400}{21} \text{ cm}^3$$

Since, Volume of the cone = [Volume of water in the cone]

∴ [Volume of water in the cone] $= \frac{4400}{21} \text{ cm}^3$

Now, Total volume of lead shots $= \frac{1}{4}$ of [Volume of water in the cone]

$$= \frac{1}{4} \times \frac{4400}{21} \text{ cm}^3 = \frac{1100}{21} \text{ cm}^3$$

Since, radius of a lead shot (sphere) (r) = 0.5 cm

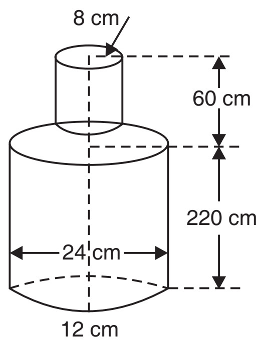
∴ Volume of 1 lead shot $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times \frac{5}{10} \times \frac{5}{10} \times \frac{5}{10} \text{ cm}^3$

∴ Number of lead shots $= \frac{\text{Total volume of lead shots}}{\text{Volume of 1 lead shot}} = \frac{\left[\frac{1100}{21}\right]}{\left[\frac{4 \times 22 \times 5 \times 5 \times 5}{3 \times 7 \times 1000}\right]}$

$$= \frac{1100}{21} \times \frac{3 \times 7 \times 1000}{4 \times 22 \times 5 \times 5 \times 5} = 100$$

Thus, the required number of lead shots = 100.

Q. 6. *A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm³ of iron has approximately 8 g mass. (Use π = 3.14)* [CBSE 2012]



Sol. Height of the big cylinder (h) = 220 cm

$$\text{Base radius } (r) = \frac{24}{2}\text{ cm} = 12\text{ cm}$$

∴ Volume of the big cylinder = $\pi r^2 h = \pi\,(12)^2 \times 220\text{ cm}^3$

Also, height of smaller cylinder (h_1) = 60 cm

Base radius (r_1) = 8 cm

∴ Volume of the smaller cylinder $\pi r_1^2 h_1 = \pi\,(8)^2 \times 60\text{ cm}^3$

∴ Volume of iron

= [Volume of big cylinder] + [Volume of the smaller cylinder]

$$= \pi \times 220 \times 12^2 + \pi \times 60 \times 8^2\text{ cm}^3$$

$$= 3.14\,[220 \times 12 \times 12 + 60 \times 8 \times 8]\text{ cm}^3 = \frac{314}{100}\,[20 \times 144 + 60 \times 64]\text{ cm}^3$$

$$= \frac{314}{100}\,[31680 + 3840]\text{ cm}^3 = \frac{314}{100} \times 35520\text{ cm}^3$$

$$\text{Mass of iron} = \frac{8 \times 314 \times 35520}{100}\text{ g} = \frac{89226240}{100}\text{ g} = \frac{8922624}{10000}\text{ g}$$

= 892.2624 kg = **892.26 kg.**

Q. 7. *A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.*

Sol. Height of the conical part = 120 cm.

Base radius of the conical part = 60 cm.

∴ Volume of the conical part = $\frac{1}{3} \times \frac{22}{7} \times 60^2 \times 120\text{ cm}^3$

Radius of the hemispherical part = 60 cm.

∴ Volume of the hemispherical part = $\frac{2}{3} \times \frac{22}{7} \times 60^3\text{ cm}^3$

∴ Volume of the solid

= [Volume of conical part] + [Volume of hemispherical part]

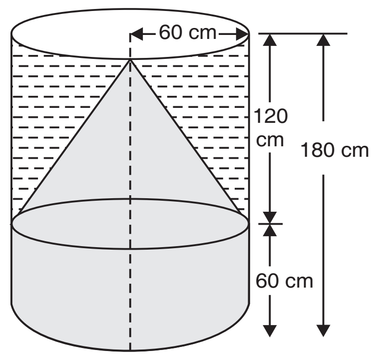
$$= \left[\frac{1}{3} \times \frac{22}{7} \times 60^2 \times 120\right] + \left[\frac{2}{3} \times \frac{22}{7} \times 60^3\right]\text{cm}^3 = \frac{2}{3} \times \frac{22}{7} \times 60^2\,[60 + 60]\text{ cm}^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times 60 \times 60 \times 120\text{ cm}^3 = \frac{2 \times 22 \times 60 \times 60 \times 40}{7}\text{cm}^3 = \frac{6336000}{7}\text{cm}^3$$

Volume of the cylinder = $\pi r^2 h$

$= \frac{22}{7} \times 60^2 \times 180 \text{ cm}^3 = \frac{22 \times 60 \times 60 \times 180}{7} \text{cm}^3$

$= \frac{14256000}{7} \text{cm}^3$



$\Rightarrow$ Volume of water in the cylinder $= \frac{14256000}{7} \text{cm}^3$

$\therefore$ Volume of the water left in the cylinder

$= \left[\frac{14256000}{7} - \frac{6336000}{7}\right] \text{cm}^3 = \frac{7920000}{7} \text{cm}^3$

$= 1131428.57142 \text{ cm}^3$

$= \frac{1131428.57142}{1000000} \text{m}^3$ [$\because$ 1000000 cm^3 = 1 m^3]

$= 1.13142857142 \text{ m}^3$ = **1.131 m^3 (approx).**

Q. 8. *A spherical glass vessel has a cylindrical neck 8 cm long, 2 cm in diameter; the diameter of the spherical part is 8.5 cm. By measuring the amount of water it holds, a child finds its volume to be 345 cm^3. Check whether she is correct, taking the above as the inside measurements, and $\pi = 3.14$.*

Sol. Volume of the cylindrical part

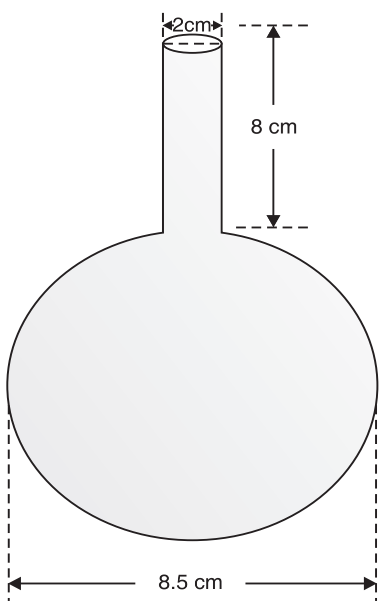
$= \pi r^2 h = 3.14 \times 1^2 \times 8 \text{ cm}^3$ [$\because$ *Radius* $= \frac{2}{2} = 1$ *cm, height (h) = 8 cm*]

$= \frac{314}{100} \times 8 \text{ cm}^3$

Volume of the spherical part

$= \frac{4}{3} \pi r_1^3$ | *Here* $r_1 = \frac{8.5}{2}$ *cm*

$= \frac{4}{3} \times \frac{314}{100} \times \frac{85}{20} \times \frac{85}{20} \times \frac{85}{20} \text{cm}^3$



Total volume of the glass-vessel

$= \left[\frac{314}{100} \times 8\right] + \left[\frac{314}{100} \times \frac{4}{3} \times \frac{85 \times 85 \times 85}{8000}\right]$

$= \frac{314}{100}\left[8 + \frac{4 \times 85 \times 85 \times 85}{24000}\right] \text{cm}^3$

$= \frac{314}{100}\left[8 + \frac{614125}{6000}\right] \text{cm}^3$

$= \frac{314}{100}\left[\frac{48000 + 614125}{6000}\right] \text{cm}^3 = \frac{314}{100}\left[\frac{662125}{6000}\right] \text{cm}^3$

$= \frac{314}{100} \times \frac{5297}{48} \text{cm}^3 = \frac{157}{100} \times \frac{5297}{24} = \frac{831629}{2400} \text{cm}^3$

$= 346.51 \text{ cm}^3$ (approx.)

$\Rightarrow$ Volume of water in the vessel = 346.51 cm^3

Since, the child finds the volume as 345 cm^3

$\therefore$ The child's answer is not correct

$\Rightarrow$ The correct answer is 346.51 cm^3.

- **Conversion of Solids from One Shape to Another**

  We generally come across situations where objects are converted from one shape to another, or a liquid, originally filled in a container of a particular shape is poured into another container of a different shape or size. In such situations the volume of the material remains the same.

NCERT TEXTBOOK QUESTIONS SOLVED

EXERCISE 13.3

[Take $\pi = \frac{22}{7}$, (Unless stated otherwise)]

Q. 1. *A metallie sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.*

Sol. Radius of the sphere (r_1) = 4.2 cm

$\therefore$ Volume of the sphere $\left(\frac{4}{3}\pi r_1^3\right) = \frac{4}{3} \times \frac{22}{7} \times \frac{42}{10} \times \frac{42}{10} \times \frac{42}{10}$ cm^3

Radius of the cylinder (r_2) = 6 cm

Let 'h' be the height of the cylinder

$\therefore$ Volume of the cylinder $= \pi r^2 h = \frac{22}{7} \times 6 \times 6 \times h$ cm^3

Since, Volume of the metallic sphere = Volume of the cylinder.

$$\Rightarrow \frac{4}{3} \times \frac{22}{7} \times \frac{42}{10} \times \frac{42}{10} \times \frac{42}{10} = \frac{22}{7} \times 6 \times 6 \times h$$

$$\Rightarrow h = \frac{4}{3} \times \frac{22}{7} \times \frac{42}{10} \times \frac{42}{10} \times \frac{42}{10} \times \frac{7}{22} \times \frac{1}{6} \times \frac{1}{6} \text{ cm}$$

$$= \frac{4 \times 7 \times 7 \times 4}{10 \times 10 \times 10} \text{ cm} = \frac{2744}{1000} \text{ cm} = \mathbf{2.744 \text{ cm.}}$$

Q. 2. *Metallic spheres of radii 6 cm, 8 cm and 10 cm, respectively, are melted to form a single solid sphere. Find the radius of the resulting sphere.*

Sol. Radii of the given spheres are:

r_1 = 6 cm

r_2 = 8 cm

r_3 = 10 cm

$\Rightarrow$ Volume of the given spheres are:

$V_1 = \frac{4}{3}\pi r_1^3$, $V_2 = \frac{4}{3}\pi r_2^3$ and $V_3 = \frac{4}{3}\pi r_3^3$

$\therefore$ Total volume of the given spheres

$= V_1 + V_2 + V_3$

$= \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_3^3 = \frac{4}{3}\pi\left[r_1^3 + r_2^3 + r_3^3\right] = \frac{4}{3} \times \frac{22}{7} \times [6^3 + 8^3 + 10^3]\ \text{cm}^3$

$= \frac{4}{3} \times \frac{22}{7} \times [216 + 512 + 1000]\ \text{cm}^3 = \frac{4}{3} \times \frac{22}{7} \times [1728]\ \text{cm}^3$

Let the radius of the new big sphere be R.

$\therefore$ Volume of the new sphere

$= \frac{4}{3} \times \pi \times R^3 = \frac{4}{3} \times \frac{22}{7} \times R^3$

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
	3

Since, the two volume must be equal.

$\therefore \quad \frac{4}{3} \times \frac{22}{7} \times R^3 = \frac{4}{3} \times \frac{22}{7} \times 1728\ \text{cm}^3$

$\Rightarrow \quad R^3 = 1728$

$\Rightarrow \quad R^3 = 2^3 \times 2^3 \times 3^3$

$\Rightarrow \quad R^3 = (2 \times 2 \times 3)^3$

$\Rightarrow \quad R = 2 \times 2 \times 3$

$\Rightarrow \quad R = 12\ \text{cm}$

Thus, the required radius of the resulting sphere = **12 cm.**

Q. 3. *A 20 m deep well with diameter 7 m is dug and the earth from digging is evenly spread out to form a platform 22 m by 14 m. Find the height of the platform.*

Sol. Diameter of the cylindrical well = 7 m

$\Rightarrow$ Radius of the cylinder $(r) = \frac{7}{2}$ m

Depth of the well (h) = 20 m

$\therefore \quad \text{Volume} = \pi r^2 h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 20\ \text{m}^3 = 22 \times 7 \times 5\ \text{m}^3$

$\Rightarrow$ Volume of the earth taken out = $22 \times 7 \times 5\ \text{m}^3$

Now this earth is spread out to form a cuboial platform having

length = 22 m

breadth = 14 m

Let 'h' be the height of the platform.

$\therefore$ Volume of the platform = $22 \times 14 \times h\ \text{m}^3$

$\therefore \quad 22 \times 14 \times h = 22 \times 7 \times 5$

$\Rightarrow \quad h = \frac{22 \times 7 \times 5}{22 \times 14} = \frac{5}{2}\ \text{m} = 2.5\ \text{m}$

Thus, the required height of the platform is **2.5 m.**

Q. 4. *A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.* [(CBSE Sample Paper 2011) CBSE 2012]

Sol. Diameter of cylindrical well (d) = 3 m

$\Rightarrow$ Radius of the cylindrical well (r) = $\frac{3}{2}$ m = 1.5 m

Depth of the well (h) = 14 m

$$\therefore \quad \text{Volume} = \pi r^2 h = \frac{22}{7} \times \left(\frac{15}{10}\right)^2 \times 14 \text{ m}^3$$

$$= \frac{22 \times 15 \times 15 \times 14}{7 \times 10 \times 10} \text{ m}^3 = \mathbf{11 \times 3 \times 3 \text{ m}^3} = 99 \text{ m}^3$$

Let the height of the embankment = 'H' metre.

Internal radius of the embankment (r) = 1.5 m.

External radius of the embankment R = (4 + 1.5) m = 5.5 m.

$\therefore$ Volume of the embankment

$$= \pi R^2 H - \pi r^2 H = \pi H [R^2 - r^2] = \pi H (R + r)(R - r)$$

$$= \frac{22}{7} \times H (5.5 + 1.5)(5.5 - 1.5) = \frac{22}{7} \times H \times 7 \times 4 \text{ m}^3$$

Since, Volume of the embankment = Volume of the cylindrical well

$$\therefore \quad \left[\frac{22}{7} \times H \times 7 \times 4\right] = 99$$

$$\Rightarrow \quad H = 99 \times \frac{7}{22} \times \frac{1}{7} \times \frac{1}{4} \text{ m} = \frac{9}{8} \text{ m} = 1.125 \text{ m}$$

Thus, the required height of the embankment = **1.125 m.**

Q. 5. *A container shaped like a right circular cylinder having diameter 12 cm and height 15 cm is full of ice cream. The ice cream is to be filled into cones of height 12 cm and diameter 6 cm, having a hemispherical shape on the top. Find the number of such cones which can be filled with ice cream.* [CBSE 2009]

Sol. **For the circular cylinder:**

Diameter = 12 cm $\Rightarrow$ Radius = $\frac{12}{2}$ = 6 cm

Height (h) = 15 cm

$\therefore$ Volume = $\pi r^2 h$

$\Rightarrow$ Volume of total ice cream = $\frac{22}{7} \times 6 \times 6 \times 15$ cm^3

For conical + hemispherical ice-cream cone:

Diameter = 6 cm $\Rightarrow$ radius (R) = 3 cm

Height of conc. (H) = 12 cm

Volume = (Volume of the conical part) + (Volume of the hemispherical part)

$$= \frac{1}{3}\pi R^2 H + \frac{2}{3}\pi R^3 = \frac{1}{3}\pi R^2 [H + 2R]$$

$$= \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \, [12 + 2 \times 3] \text{ cm}^3 = \frac{22 \times 3}{7} \times 18 \text{ cm}^3$$

Let number of ice-cream cones required to fill the total ice cream = n.

$$\therefore \quad n\left[\frac{22 \times 3}{7} \times 18\right] = \frac{22}{7} \times 6 \times 6 \times 15$$

$$\Rightarrow \quad n = \frac{22}{7} \times 6 \times 6 \times 15 \times \frac{7}{22} \times \frac{1}{3} \times \frac{1}{18}$$

$$\Rightarrow \quad n = 2 \times 5 = 10$$

Thus, the required number of cones is **10.**

Q. 6. *How many silver coins, 1.75 cm in diameter and of thickness 2 mm, must be melted to form a cuboid of dimensions 5.5 cm × 10 cm × 3.5 cm ?*

Sol. For a circular coin:

$$\text{Diameter} = 1.75 \text{ cm}$$

$$\Rightarrow \quad \text{Radius } (r) = \frac{175}{200} \text{ cm}$$

$$\text{Thickness } (h) = 2 \text{ mm} = \frac{2}{10} \text{ cm}$$

$$\therefore \quad \text{Volume} = \pi r^2 h$$

$$= \frac{22}{7} \times \left(\frac{175}{200}\right)^2 \times \frac{2}{10} \text{ cm}^3 \qquad | \because \textit{A coin is like a cylinder}$$

For a cuboid:

$$\text{Length } (l) = 10 \text{ cm}, \quad \text{Breadth } (b) = 5.5 \text{ cm}$$

and $\text{Height } (h) = 3.5 \text{ cm}$

$$\therefore \quad \text{Volume} = 10 \times \frac{55}{10} \times \frac{35}{10} \text{ cm}^3$$

Number of coins

Let the number of coins need to melt be 'n'

$$\therefore \quad n = \left[10 \times \frac{55}{10} \times \frac{35}{10}\right] \div \left[\frac{22}{7} \times \frac{175}{200} \times \frac{175}{200} \times \frac{2}{10}\right]$$

$$= 10 \times \frac{55}{10} \times \frac{35}{10} \times \frac{7}{22} \times \frac{200}{175} \times \frac{200}{175} \times \frac{10}{2} = 16 \times 25 = 400$$

Thus, the required number of coins = **400.**

Q. 7. *A cylindrical bucket, 32 cm high and with radius of base 18 cm, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.* [CBSE 2014]

Sol. For the cylindrical bucket:

$$\text{Radius } (r) = 18 \text{ cm}$$

$$\text{Height } (h) = 32 \text{ cm}$$

$$\text{Volume} = \pi r^2 h$$

$$= \frac{22}{7} \times (18)^2 \times 32 \text{ cm}^3$$

$$\Rightarrow \quad \text{Volume of the sand} = \left(\frac{22}{7} \times 18 \times 18 \times 32\right) \text{cm}^3$$

For the conical heap:

Height (H) = 24 cm

Let radius of the base be (R).

$\therefore$ Volume of conical heap $= \frac{1}{3}\pi R^2 H = \left[\frac{1}{3} \times \frac{22}{7} \times R^2 \times 24\right] \text{cm}^3$

Radius of the conical heap of sand:

$\because$ Volume of the conical heap of sand = Volume of the sand

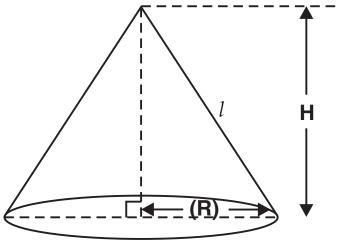
$$\therefore \quad \frac{1}{3} \times \frac{22}{7} \times R^2 \times 24 = \frac{27}{7} \times 18 \times 18 \times 32$$

$$\Rightarrow \quad R^2 = \frac{22}{7} \times 18 \times 18 \times 32 \times 3 \times \frac{7}{22} \times \frac{1}{24} = 18 \times 18 \times 4 = 18^2 \times 2^2$$

$$\Rightarrow \quad R = \sqrt{18^2 \times 2^2} = 18 \times 2 \text{ cm} = 36 \text{ cm}$$

Slant Height

Let 'l' be the slant height of the conical heap of the sand.



$$\therefore \quad l^2 = R^2 + H^2$$

$$\Rightarrow \quad l^2 = 24^2 + 36^2$$

$$\Rightarrow \quad l^2 = (12 \times 2)^2 + (12 \times 3)^2$$

$$\Rightarrow \quad l^2 = 12^2 [2^2 + 3^2]$$

$$\Rightarrow \quad l^2 = 12^2 \times 13$$

$$\Rightarrow \quad l = \sqrt{12^2 \times 13} = 12 \times \sqrt{13}$$

Thus, the required height = **36 cm** and slant height = $\mathbf{12\sqrt{13}}$ **cm.**

Q. 8. *Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm of standing water is needed ?* [CBSE 2012, AI. CBSE 2014]

Sol. Width of the canal = 6 m

Depth of the canal = 1.5 m

Length of the water column in 1 hr = 10 km

$\therefore$ Length of the water column in 30 minutes $\left(i.e., \frac{1}{2} \text{ hr}\right)$

$$= \frac{10}{2} \text{ km} = \frac{10000}{2} \text{ m} = 5000 \text{ m}$$

$\therefore$ Volume of water flown in $\frac{1}{2}$ hr

$$= 6 \times 1.5 \times 5000 \text{ m}^3 = 6 \times \frac{15}{10} \times 5000 \text{ m}^3 = 45000 \text{ m}^3$$

Since the above amount (volume) of water is spread in the form of a cuboid of height as 8 cm $\left(= \frac{8}{100} \text{ m}\right)$.

Let the area of the cuboid = a

$\therefore$ Volume of the cuboid=Area × Height $= a \times \frac{8}{100} \text{ m}^3$

Thus, $a \times \frac{8}{100} = 45000$

$\Rightarrow \quad a = \frac{45000 \times 100}{8} = \frac{4500000}{8} \text{ m}^2$

$\Rightarrow \quad a = 562500 \text{ m}^2 = \frac{562500}{10000} \text{ hectares} = 56.25 \text{ hectares}$

Thus, the required area = **56.25 hectares.**

Q. 9. *A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in her field, which is 10 m in diameter and 2 m deep. If water flows through the pipe at the rate of 3 km/h, in how much time will the tank be filled ?*

Sol. Diameter of the pipe = 20 cm

$\Rightarrow$ Radius of the pipe $(r) = \frac{20}{2}$ cm $= 10$ cm

Since, the water flows through the pipe at 3 km/hr.

$\therefore$ Length of water column per hour

$(h) = 3 \text{ km} = 3 \times 1000 \text{ m} = 3000 \times 100 \text{ cm} = 300000 \text{ cm}.$

$\therefore$ Volume of water $= \pi r^2 h = \pi \times 10^2 \times 300000 \text{ cm}^3 = \pi \times 30000000 \text{ cm}^3$

Now, for the **cylindrical tank,**

Diameter = 10 m

$\Rightarrow$ Radius $(R) = \frac{10}{2} \text{ m} = 5 \times 100 \text{ cm} = 500 \text{ cm}$

Height $(H) = 2 \text{ m} = 2 \times 100 \text{ cm} = 200 \text{ cm}$

$\therefore$ Volume of the cylindrical tank

$= \pi R^2 H = \pi \times (500)^2 \times 200 \text{ cm}^3$

Now, time required to fill the tank

$$= \frac{[\text{Volume of the tank}]}{[\text{Volume of water flown in 1 hour}]} = \frac{\pi \times 500 \times 500 \times 200}{\pi \times 30000000} \text{ hrs}$$

$$= \frac{5 \times 5 \times 2}{30} \text{ hrs} = \frac{5}{3} \text{ hrs} = \frac{5}{3} \times 60 \text{ minutes} = \textbf{100 minutes.}$$

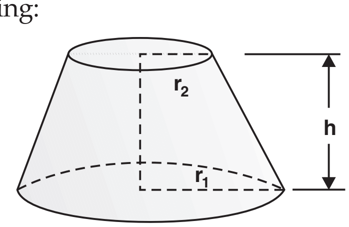
- **Frustum**

Let us consider a right circular cone, which is sliced through by a plane parallel to its base, when the smaller conical portion is removed, the resulting solid is called a **Frustum** of right circular cone. The formulae involving the frustum of a cone having:

h = the height

l = the slant height

r_1 and r_2 = the radii of the ends $(r_1 > r_2)$, are given below:



(*i*) Volume of the frustum of the cone = $\frac{1}{3} \pi h (r_1^2 + r_2^2 + r_1 r_2)$

(*ii*) Curved surface area of the frustum of the cone

$= \pi (r_1 + r_2) l$ $\quad \left[\text{where } l = \sqrt{h^2 + (r_1 - r_2)^2}\right]$

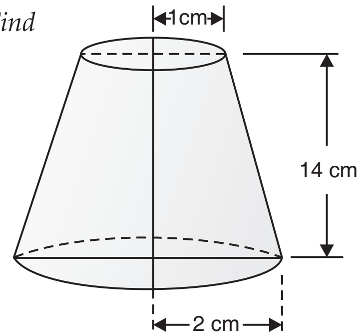
(*iii*) Total surface area of the frustum of the cone = $\pi l (r_1 + r_2) + \pi r_1^2 + \pi r_2^2$

NCERT TEXTBOOK QUESTIONS SOLVED

EXERCISE 13.4

[Use $\pi = \frac{22}{7}$, unless stated otherwise]

Q. 1. *A drinking glass is in the shape of a frustum of a cone of height 14 cm. The diameters of its two circular ends are 4 cm and 2 cm. Find the capacity of the glass.*



Sol. We have: $r_1 = 2$ cm

$r_2 = 1$ cm

$h = 14$ cm

Volume of the glass

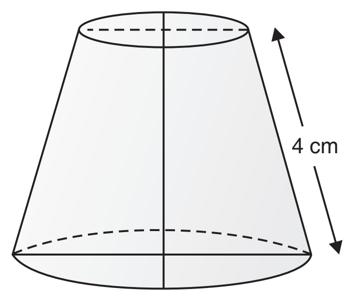
$= \frac{1}{3}\pi h (r_1^2 + r_2^2 + r_1 r_2)$

$= \frac{1}{3} \times \frac{22}{7} \times 14 \times [2^2 + 1^2 + 2 \times 1]$ cm^3

$= \frac{22}{3} \times 2\ [4 + 1 + 2]$ cm^3 $= \frac{22}{3} \times 2\ [7]$ cm^3 $= \frac{308}{3}$ cm^3 $=$ **$102\frac{2}{3}$ cm^3.**

Q. 2. *The slant height of a frustum of a cone is 4 cm and the perimeters (circumference) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.*

Sol. We have:



Slant height (l) = 4 cm

$2\pi r_1 = 18$ cm

and $2\pi r_2 = 6$ cm

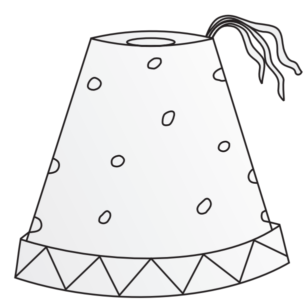
$\Rightarrow$ $\pi r_1 = \frac{18}{2} = 9$ cm

$\pi r_2 = \frac{6}{2} = 3$ cm

$\therefore$ Curved surface area of the frustum of the cone

$= \pi (r_1 + r_2)\, l = (\pi r_1 + \pi r_2)\, l = (9 + 3) \times 4$ cm^2 $= 12 \times 4$ cm^2 $=$ **48 cm^2.**

Q. 3. *A fez, the cap used by the Turks, is shaped like the frustum of a cone (see Fig.). If its radius on the open side is 10 cm, radius at the upper base is 4 cm and its slant height is 15 cm, find the area of material used for making it.* [CBSE 2012]



Sol. Here, the radius of the open side (r_1) = 10 cm

The radius of the upper base (r_2) = 4 cm

Slant height (l) = 15 cm

$\therefore$ Area of the material required

= [Curved surface area of the frustum] + [Area of the top end]

$= \pi (r_1 + r_2)\, l + \pi r_2^2$

$= \frac{22}{7} \times (10 + 4) \times 15 + \frac{22}{7} \times 4 \times 4$

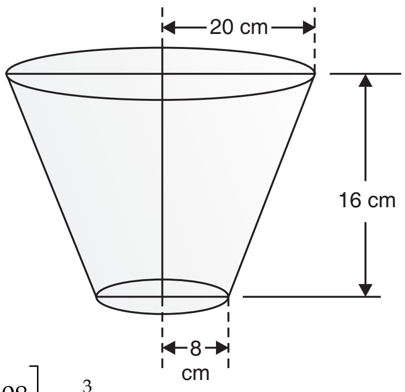
$$= \left[\frac{22}{7} \times 14 \times 15 + \frac{22}{7} \times 16\right] \text{cm}^2 = \left[(22 \times 2 \times 15) + \left(\frac{22 \times 16}{7}\right)\right] \text{cm}^2$$

$$= 660 + \frac{352}{7} \text{cm}^2 = \frac{4620 + 352}{7} \text{cm}^2 = \frac{4972}{7} \text{cm}^2 = \mathbf{710\frac{2}{7}\ cm^2}.$$

Q. 4. *A container, opened from the top and made up of a metal sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm, respectively. Find the cost of the milk which can completely fill the container, at the rate of ₹ 20 per litre. Also find the cost of metal sheet used to make the container, if it costs ₹ 8 per 100 cm². (Take $\pi = 3.14$)* *[AI. CBSE 2014]*

Sol. We have: $r_1 = 20$ cm, $r_2 = 8$ cm

and $h = 16$ cm



∴ Volume of the frustum

$$= \frac{1}{3}\pi h\ [r_1^2 + r_2^2 + r_1 r_2]$$

$$= \frac{1}{3} \times \frac{314}{100} \times 16\ [20^2 + 8^2 + 20 \times 8]\ \text{cm}^2$$

$$= \frac{1}{3} \times \frac{314}{100} \times 16\ [400 + 64 + 160]\ \text{cm}^2$$

$$= \frac{1}{3} \times \frac{314}{100} \times 16 \times 624\ \text{cm}^3 = \left[\frac{314}{100} \times 16 \times 208\right] \text{cm}^3$$

$$= \left[\frac{314}{100} \times 16 \times 208\right] \div 1000 \text{ litres} = \frac{314 \times 16 \times 208}{100000} \text{ litres}$$

$$\therefore \text{ Cost of milk} = ₹\ 20 \times \frac{314 \times 16 \times 208}{100000}$$

$$= ₹\ \frac{628 \times 16 \times 208}{10000} = ₹\ \frac{2089984}{10000} = ₹\ 208.998 \approx \mathbf{₹\ 209.}$$

Now, slant height of the given frustum

$$l = \sqrt{h^2 + (r_1 - r_2)^2} = \sqrt{16^2 + (20-8)^2} = \sqrt{16^2 + 12^2}$$

$$= \sqrt{256 + 144} = \sqrt{400} = \mathbf{20\ cm}$$

∴ Curved surface area

$$= \pi\ (r_1 + r_2)\ l = \frac{314}{100}\ (20 + 8) \times 20\ \text{cm}^2$$

$$= \frac{314}{100} \times 28 \times 20\ \text{cm}^2 = \frac{314}{5} \times 28\ \text{cm}^2 = \frac{8792}{5} \text{cm}^2 = \mathbf{1758.4\ cm^2}$$

Area of the bottom

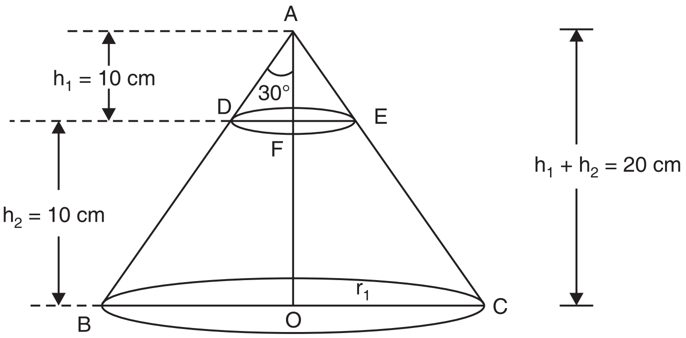
$$= \pi r^2 = \frac{314}{100} \times 8 \times 8\ \text{cm}^2 = \frac{20096}{100} \text{cm}^2 = \mathbf{200.96\ cm^2}$$

∴ Total area of metal required = 1758.4 cm² + 200.96 cm² = 1959.36 cm²

$$\text{Cost of metal required} = ₹\ \frac{8}{100} \times 1959.36 = \mathbf{₹\ 156.75.}$$

Q. 5. *A metallic right circular cone 20 cm high and whose vertical angle is 60° is cut into two parts at the middle of its height by a plane parallel to its base. If the frustum so obtained be drawn into a wire of diameter $\frac{1}{16}$ cm, find the length of the wire.* [CBSE 2012]

Sol. Let us consider the frustum *DECB* of the metallic cone *ABC*



Here, $r_1 = BO$ and $r_2 = DF$

In Δ *AOB*, $\frac{r_1}{(h_1+h_2)} = \tan 30° = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad r_1 = (h_1 + h_2) \times \frac{1}{\sqrt{3}} = 20 \times \frac{1}{\sqrt{3}}$

In Δ *ADF*, $\frac{r_2}{h_1} = \tan 30°$

$\Rightarrow \quad r_2 = h_1 \times \frac{1}{\sqrt{3}} = 10 \times \frac{1}{\sqrt{3}}$

Now, the volume of the frustum *DBCE*

$$= \frac{1}{3}\pi h_2 \, [r_1^2 + r_2^2 + r_1 r_2] = \frac{1}{3} \times \pi \times 10\left[\left(\frac{20}{\sqrt{3}}\right)^2 + \left(\frac{10}{\sqrt{3}}\right)^2 + \frac{20}{\sqrt{3}} \times \frac{10}{\sqrt{3}}\right]$$

$$= \frac{\pi}{3} \times 10\left[\frac{400}{3} + \frac{100}{3} + \frac{200}{3}\right] = \frac{\pi}{3} \times 10\left[\frac{700}{3}\right]$$

Let l be the length and D be diameter of the wire drawn from the frustum. Since the wire is in the form of a cylinder,

∴ Volume of the wire $= \pi r^2 l$

$$= \pi\left(\frac{D}{2}\right)^2 \times l = \frac{\pi D^2 l}{4} = \frac{\pi l}{4 \times 16 \times 16} \qquad \left|\because D = \frac{1}{16}\right.$$

∵ [Volume of the frustum] = [Volume of the wire]

$$\therefore \quad \left[\frac{\pi}{3} \times 10 \times \frac{700}{3}\right] = \frac{\pi l}{4 \times 16 \times 16}$$

$$\Rightarrow \quad \frac{l}{4 \times 16 \times 16} = \frac{10 \times 700}{3 \times 3}$$

$$\Rightarrow \quad l = \frac{10 \times 700}{3 \times 3} \times 4 \times 16 \times 16 = \frac{7168000}{9 \times 100} = 7964.44 \text{ m}$$

Thus, the required length of the wire = **7964.44 m.**

NCERT TEXTBOOK QUESTIONS SOLVED

EXERCISE 13.5

Q. 1. *A copper wire, 3 mm in diameter, is wound about a cylinder whose length is 12 cm, and diameter 10 cm, so as to cover the curved surface of the cylinder. Find the length and mass of the wire, assuming the density of copper to be 8.88 g per cm^3.*

Sol. Since, diameter of the cylinder = 10 cm

$\therefore$ Radius of the cylinder (r) = $\frac{10}{2}$ cm = 5 cm

$\Rightarrow$ Length of wire in completely one round

$$2\pi r = 2 \times 3.14 \times 5 \text{ cm} = 31.4 \text{ cm}$$

$\because$ Diameter of wire = 3 mm = $\frac{3}{10}$ cm

$\therefore$ The thickness of cylinder covered in one round = $\frac{3}{10}$ cm

$\Rightarrow$ Number of rounds (turns) of the wire to cover 12 cm

$$= \frac{12}{3/10} = 12 \times \frac{10}{3} = 40$$

$\therefore$ Length of wire required to cover the whole surface = Length of wire required to complete 40 rounds

$$= 40 \times 31.4 \text{ cm} = \mathbf{1256 \text{ cm}}$$

Now, radius of the wire = $\frac{3}{2}$ mm = $\frac{3}{20}$ cm

$\therefore$ Volume of wire = $\pi r^2 l$

$$= \frac{22}{7} \times \frac{3}{20} \times \frac{3}{20} \times 1256 \text{ cm}^3$$

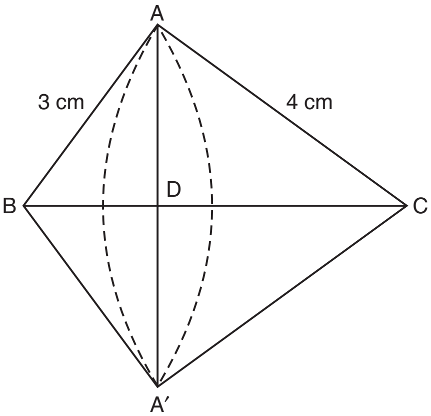
$\because$ Density of wire = 8.88 gm/cm^3

$\therefore$ Weight of the wire = [Volume of the wire] × Density

$$= \left[\frac{22}{7} \times \frac{3}{20} \times \frac{3}{20} \times 1256\right] \times 8.88 \text{ gm} = \frac{22}{7} \times \frac{3}{20} \times \frac{3}{20} \times 1256 \times \frac{888}{100} \text{ gm}$$

$$= \mathbf{788 \text{ g (approx.)}}$$

Q. 2. *A right triangle, whose sides are 3 cm and 4 cm (other than hypotenuse) is made to revolve about its hypotenuse. Find the volume and surface area of the double cone so formed. Choose value of π as found appropriate.* [CBSE 2012]



Sol. Let us consider the rt ΔBAC, rt. angled at A such that

$$AB = 3 \text{ cm}, \quad AC = 4 \text{ cm}$$

$\therefore$ Hypotenuse $BC = \sqrt{3^2 + 4^2} = 5$ cm

Obviously, we have obtained two cones on the same base AA' such that the radius = DA or DA'

Now, $\dfrac{AD}{CA} = \dfrac{AB}{CB}$ $\quad | \because ADB \sim \Delta CAB$

$\Rightarrow \dfrac{AD}{4} = \dfrac{3}{5} \Rightarrow AD = \dfrac{3}{5} \times 4 = \dfrac{12}{5}$ cm

Also, $\dfrac{DB}{AB} = \dfrac{AB}{CB}$

$\Rightarrow \dfrac{DB}{3} = \dfrac{3}{5} \Rightarrow DB = \dfrac{3 \times 3}{5} = \dfrac{9}{5}$ cm

Since, $CD = BC - DB \Rightarrow CD = 5 - \dfrac{9}{5} = \dfrac{16}{5}$ cm

Now, volume of the double cone

$$= \left[\frac{1}{3}\pi \times \left(\frac{12}{5}\right)^2 \frac{9}{5} + \frac{1}{3}\pi \times \left(\frac{12}{5}\right)^2 \times \frac{16}{5}\right] \text{cm}^3 = \frac{1}{3}\pi \times \left(\frac{12}{5}\right)^2 \left[\frac{9}{5} + \frac{16}{5}\right] \text{cm}^3$$

$$= \frac{1}{3}\pi \times \frac{144}{25} \times 5 \text{ cm}^3 = \frac{1}{3} \times \frac{314}{100} \times \frac{144}{25} \times 5 \text{ cm}^3 = \mathbf{30.14 \text{ cm}^3}$$

Surface area of the double cone

$$= \left(\pi \times \frac{12}{5} \times 3\right) + \left(\pi \times \frac{12}{5} \times 4\right) \text{cm}^2 = \pi \times \frac{12}{5} [3 + 4] \text{ cm}^2$$

$$= \frac{314}{100} \times \frac{12}{5} \times 7 \text{ cm}^2 = \mathbf{52.75 \text{ cm}^2}$$

Q. 3. *A cistern, internally measuring 150 cm × 120 cm × 110 cm, has 129600 cm³ of water in it. Porous bricks are placed in the water until the cistern is full to the brim. Each brick absorbs one-seventeenth of its own volume of water. How many bricks can be put in without overflowing the water, each being 22.5 cm × 7.5 cm × 6.5 cm?*

Sol. ∵ Dimensions of the cistern are 150 cm, 120 cm and 110 cm.

∴ Volume of the cistern = $150 \times 120 \times 110 \text{ cm}^3 = 1980000 \text{ cm}^3$

Volume of water contained in the cistern = 129600 cm^3

∴ Free space (volume) which is not filled with water

$$= 1980000 - 129600 \text{ cm}^3 = 1850400 \text{ cm}^3$$

Now, Volume of one brick = $22.5 \times 7.5 \times 6.5 \text{ cm}^3$

$$= \frac{225}{10} \times \frac{75}{10} \times \frac{65}{10} \text{ cm}^2 = 1096.875 \text{ cm}^3$$

∴ Volume of water absorbed by one brick = $\frac{l}{17}(1096.875) \text{ cm}^3$

Let 'n' bricks can be put in the cistern.

∴ Volume of water absorbed by 'n' bricks= $\frac{n}{17}(1096.875)$

∴ [Volume occupied by 'n' bricks] = [(free space in the cistern) + (volume of water absorbed by n-bricks)]

$$\Rightarrow [n \times (1096.875)] = \left[1850400 + \frac{n}{17}(1096.875)\right]$$

$$\Rightarrow 1096.875\, n - \frac{n}{17}(1096.875) = 1850400$$

$$\Rightarrow \left(n - \frac{n}{17}\right) \times 1096.875 = 1850400$$

$$\Rightarrow \frac{16}{17}n = \frac{1850400}{1096.875}$$

$$\Rightarrow n = \frac{1850400 \times 1000}{1096875} \times \frac{17}{16} = 1792.4102 \approx 1792$$

Thus, **1792 bricks** can be put in the cistern.

Q. 4. *In one fortnight of a given month, there was a rainfall of 10 cm in a river valley. If the area of the valley is 97280 km², show that the total rainfall was approximately equivalent to the addition to the normal water of three rivers each 1072 km long, 75 m wide and 3 m deep.*

Sol. Volume of three rivers = 3 {(Surface area of a river) × Depth}

$$= 3\left\{\left(1072 \text{ km} \times \frac{75}{1000} \text{ km}\right) \times \frac{3}{1000} \text{ km}\right\} = 3\left\{\frac{241200}{1000000} \text{ km}^3\right\}$$

$$= \frac{723600}{1000000} \text{ km}^3 = \mathbf{0.7236 \text{ km}^3}$$

Volume of rainfall = (surface area) × (height of rainfall)

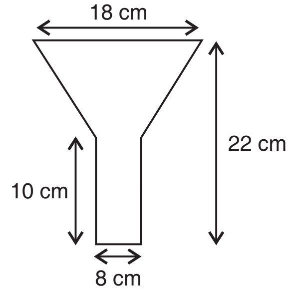
$$= 97280 \times \frac{10}{100 \times 1000} \text{ km}^3 \qquad \left[\because 10 \text{ cm} = \frac{10}{100 \times 1000} \text{ km}\right]$$

$$= \frac{9728}{1000} \text{ km}^3 = \mathbf{9.728 \text{ km}^3}$$

Since, $0.7236 \text{ km}^3 \neq 9.728 \text{ km}^3$

∴ The additional water in the three rivers is not equivalent to the rainfall.

Q. 5. *An oil funnel made of tin sheet consists of a 10 cm long cylindrical portion attached to a frustum of a cone. If the total height is 22 cm, diameter of the cylindrical portion is 8 cm and the diameter of the top of the funnel is 18 cm, find the area of the tin sheet required to make the funnel (see Fig.).*



Sol. We have, for the **cylindrical part**

Diameter = 8 cm

$\Rightarrow$ Radius (r) = 4 cm

Height = 10 cm

$\Rightarrow$ Curved Surface area $= 2\pi rh = 2 \times \frac{22}{7} \times 4 \times 10 \text{ cm}^2 = \frac{22}{7} \times 80 \text{ cm}^2$

For the frustum: $r_1 = \frac{18}{2}$ cm = 9 cm

and $r_2 = \frac{8 \text{ cm}}{2} = 4$ cm

Height (H) = 22 – 10 = 12 cm

$\therefore$ Slant height $(l) = \sqrt{H^2 + (r_1 - r_2)^2} = \sqrt{12^2 + (9-4)^2}$

$= \sqrt{144 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13$

$\therefore$Surface area, $\pi (r_1 + r_2)\, l = \frac{22}{7} \times (4 + 9) \times 13 \text{ cm}^2$

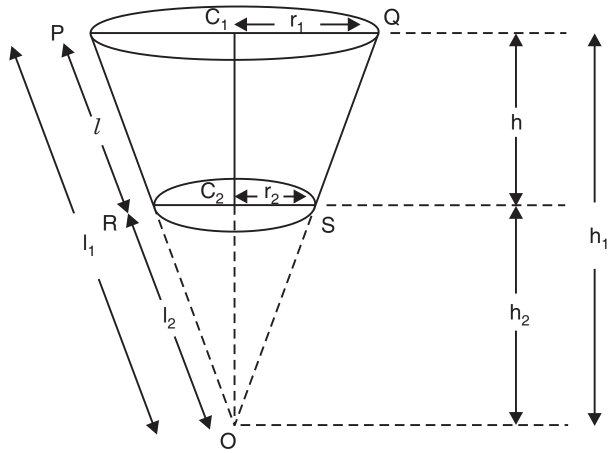
$= \frac{22}{7} \times 13 \times 13 \text{ cm}^2 = \frac{22}{7} \times 16.9 \text{ cm}^2$

Area of tin required = [Area of the frustum] + [Area of cylindrical portion]

$= \frac{22}{7} \times 169 \text{ cm}^2 + \frac{22}{7} \times 80 \text{ cm}^2 = \frac{22}{7} (169 + 80) \text{ cm}^2$

$= \frac{22}{7} \times 249 \text{ cm}^2 = \frac{5478}{7} \text{cm}^2 = \mathbf{782 \frac{4}{7} \text{ cm}^2}$

Q. 6. *Derive the formula for the curved surface area and total surface area of the frustum of a cone, given to you in Section 13.5, using the symbols as explained.*



Sol. We have,

Curved surface area of the frustum *PQRS*

$$= \begin{bmatrix}\text{curved surface area}\\ \text{of the rt circular}\\ \text{cone } OPQ\end{bmatrix} - \begin{bmatrix}\text{curved surface area}\\ \text{of the rt circular}\\ \text{cone } ORS\end{bmatrix}$$

$$= \pi r_1 l_1 - \pi r_2 l_2 \qquad \text{...(1)}$$

Now, $\Delta OC_1 Q \sim \Delta OC_2 S$

$$\therefore \quad \frac{OQ}{OS} = \frac{QC_1}{SC_2} = \frac{OC_1}{OC_2}$$

$$\Rightarrow \quad \frac{l_1}{l_2} = \frac{r_1}{r_2} = \frac{h_1}{h_2}$$

$$\Rightarrow \quad \frac{l + l_2}{l_2} = \frac{r_1}{r_2} \qquad | \because l_1 = l + l_2$$

$$\Rightarrow \quad \frac{l}{l_2} + 1 = \frac{r_1}{r_2} \Rightarrow \frac{l}{l_2} = \frac{r_1}{r_2} - 1$$

$$\therefore \quad l = \left(\frac{r_1 - r_2}{r_2}\right) l_2 \qquad \text{...(2)}$$

Now, from (1),

curved surface area of the frustum

$$= \pi r_1 \left(\frac{r_1}{r_2} l_2\right) - \pi r_2 l_2 = \pi l_2 \left[\frac{r_1^2}{r_2} - r_2\right] = \pi l_2 \left(\frac{r_1^2 - r_2^2}{r_2}\right)$$

$$= \pi l_2 \left[\frac{(r_1 + r_2)(r_1 - r_2)}{r_2}\right] = \pi \left(\frac{r_1 - r_2}{r_2}\right) l_2 \times (r_1 + r_2)$$

$$= \pi l (r_1 + r_2) \qquad | \because \left(\frac{r_1 - r_2}{r_2}\right) l_2 = l \text{ From (2)}$$

Now, the total surface area of the frustum

= (curved surface area) + (base surface area) + (top surfaces area)

$$= \pi l (r_1 + r_2) + \pi r_2^2 + \pi r_1^2 = \pi (r_1 + r_2) l + \pi (r_1^2 + r_2^2)$$

$$= \boldsymbol{\pi [(r_1 + r_2) l + r_1^2 + r_2^2]}$$

Q. 7. *Derive the formula for the volume of the frustum of a cone, given to you in Section 13.5, using the symbols as explained.*

Sol. We have,

[Volume of the frustum *RPQS*]

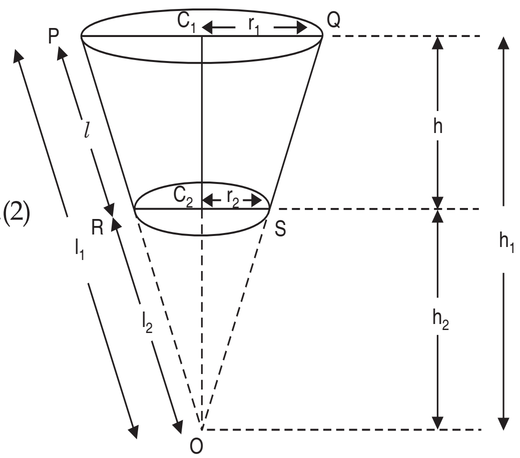
= [Volume of right circular cone *OPQ*] – [Volume of right circular cone *ORS*]

$$= \frac{1}{3}\pi r_1^2 h_1 - \frac{1}{3}\pi r_2^2 h_2$$

$$= \frac{1}{3} \pi [r_1^2 h - r_2^2 h_2] \qquad \text{...(1)}$$

Since $\Delta OC_1Q \sim \Delta OC_2S$

$$\therefore \quad \frac{OQ}{OS} = \frac{QC_1}{SC_2} = \frac{OC_1}{OC_2}$$



$$\Rightarrow \quad \frac{l_1}{l_2} = \frac{r_1}{r_2} = \frac{h_1}{h_2} \Rightarrow h_1 = \left(\frac{r_1}{r_2} \times h_2\right) \quad ...(2)$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{h+h_2}{h_2}$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{h}{h_2}+1$$

$$\Rightarrow \quad \frac{h}{h_2} = \frac{r_1}{r_2}-1 \Rightarrow h = \left[\frac{r_1}{r_2}-1\right] \times h_2$$

$$\Rightarrow \quad h = (r_1 - r_2)\frac{h_2}{r_2} \quad ...(3)$$

From (1) and (2), we have

$$\{\text{volume of the frustum } RPQS\} = \frac{1}{3}\pi\left[r_1^2 \times \frac{r_1}{r_2}h_2 - r_2^2\, h_2\right]$$

$$= \frac{1}{3}\pi\left[\frac{r_1^3}{r_2} - r_2^2\right]h_2$$

$$= \frac{1}{3}\pi\,[r_1^3 - r_2^3]\frac{h_2}{r_2}$$

$$= \frac{1}{3}\pi\,(r_1^2 + r_2^2 + r_1\, r_2)\left[(r_1 - r_2)\frac{h_2}{r_2}\right]$$

$$= \mathbf{\frac{1}{3}\pi\,(r_1^2 + r_2^2 + r_1 r_2)\; h} \quad | \textit{ From } (3)$$

MORE QUESTIONS SOLVED

I. VERY SHORT ANSWER TYPE QUESTIONS

Q. 1. *A cylinder, a cone and a hemisphere have the same values for r and h. Find the ratio of their volumes.*

Sol. Let radius be 'r' and height 'h'.

$\therefore \quad r = h$

[Volume of a cylinder] : [Volume of a cone] : [Volume of a hemisphere]

$$\Rightarrow \quad \pi r^2 h \; : \; \frac{1}{3}\pi r^2 h \; : \; \frac{2}{3}\pi r^3$$

$$\Rightarrow \quad h \; : \; \frac{1}{3}h \; : \; \frac{2}{3}r \quad | \text{ Dividing by } \pi r^2$$

$$\Rightarrow \quad h \quad : \quad \frac{1}{3}h \quad : \quad \frac{2}{3}h \qquad | \because r = h$$

$$\Rightarrow \quad 1 \quad : \quad \frac{1}{3} \quad : \quad \frac{2}{3} \qquad | \text{ Dividing by } h$$

$$\Rightarrow \quad 3 \quad : \quad 1 \quad : \quad 2 \qquad | \text{ multiplying by } 3$$

$\therefore$ The required ratio = 3 : 1 : 2

Q. 2. *If two identical solid cubes of side 'x' are joined end to end, then the total surface area of the resulting cuboid is $12x^2$. Is it true?*

Sol. $\because$ The total surface area of a cube of side x is $6x^2$

When they are joined end to end, the lenght bcomes $2x$

$$\therefore \quad \text{Total surface area} = 2[lh + bh + hl] = 2\,[(2x \cdot x) + (x \cdot x) + (2x \cdot x)]$$
$$= 2\,[2x^2 + x^2 + 2x^2] = 2\,[5x^2] = 10x^2 \neq 12x^2$$

$\therefore$ False

Q. 3. *A spherical ball is melted to make eight new identical balls. Then the radius of each new ball is $\frac{1}{8}th$ of the radius of the original ball. Is it true?*

Sol. $\because$ Radius 'R' of original ball $\Rightarrow$ volume = $\frac{4}{3}\pi R^3$

And Radius 'r' of the new ball $\Rightarrow$ volume = $\frac{4}{3}\pi r^3$

$$\therefore \quad \frac{4}{3}\pi R^3 = 8\left[\frac{4}{3}\pi r^3\right]$$

$$\Rightarrow \quad R^3 = (2r)^3 \quad \Rightarrow \quad R = 2r \quad \text{or} \quad r = \frac{1}{2}R$$

$\Rightarrow$ Radius of the new ball = Half the radius of original ball.

$\therefore$ False

Q. 4. *If a solid cone of base radius 'r' and height 'h' is placed over a solid cylinder having same base radius 'r' and height - 'h' as that of the cone, then the curved surface area of the shape is $\pi r\left(\sqrt{h^2+r^2}\right) + 2\,\pi rh$. Is it true?*

Sol. $\because$ Curved surface area of a cone $= \pi rl = \pi r\sqrt{h^2+r^2}$ $\qquad | \because l = \sqrt{r^2+h^2}$

And curved surface area of the cylinder = $2\pi rh$

$\therefore$ The curved surface area of the combination = $\pi r\left(\sqrt{h^2+r^2}\right) + 2\pi rh$

$\therefore$ True.

Q. 5. *A cylinder and a cone are of the same base radius and same height. Find the ratio of the volumes of the cylinder of that of the cone.* (CBSE 2009)

Sol. Let the base radius = r and height = h

$$\therefore \quad \frac{\text{Volume of cylinder}}{\text{Volume of cone}} = \frac{\pi r^2 h}{\frac{1}{3}\pi r^2 h} = \frac{1}{1/3} = 1 \times \frac{3}{1} = \frac{3}{1}$$

$\Rightarrow$ The required ratio = 3 : 1

II. SHORT ANSWER TYPE QUESTIONS

Q. 1. *A metallic sphere of radius 4.2 cm is melted and recast into the shape of a cylinder of radius 6 cm. Find the height of the cylinder.*

Sol. Here, radius (r) = 4.2 cm

$\therefore$ Volume of the sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi r^3 \left(\frac{42}{10}\right)^3 \text{cm}^3$

Let the height of the cylinder = h

$\because$ Radius of the cylinder (R) = 6 cm

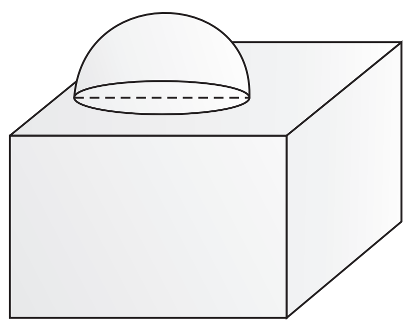
$\therefore$ Volume of the cylinder $= \pi R^2 h = \pi \times (6)^2 \times h$

Since, [Volume of the cylinder] = [Volume of the sphere]

$$\therefore \quad \pi \times 6^2 \times h = \frac{4}{3} \times \pi \times \frac{42}{10} \times \frac{42}{10} \times \frac{42}{10}$$

$$\Rightarrow \quad h = \frac{4}{3} \times \frac{\pi}{\pi} \times \frac{42 \times 42 \times 42}{1000} \times \frac{1}{36} = \frac{2744}{1000} \text{ cm} = \mathbf{2.744 \text{ cm}}$$

Q. 2. *The figure shows a decorative block which is made of two solids—a cube and a hemisphere. The base of the block is a cube with edge 5 cm and the hemisphere, fixed on the top, has a diameter of 4.2 cm. Find the total surface area of the block.* [*Take* $\pi = \frac{22}{7}$] (AI CBSE 2009)



Sol. $\because$ Total surface area of a cube $= 6 \times (\text{side})^2$

$\therefore$ Total surface area of the cubical block

$= 6 \times (5)^2 -$ [Base area of the hemisphere]

$= 150 - \pi r^2$

Now, total surface area of the solid

$= 150 - \pi r^2 + 2\pi r^2 = 150 + \pi r^2$

$= 150 \times \frac{22}{7} \times \frac{21}{10} \times \frac{21}{10}$ $\qquad \because d = 4.2$

$\qquad \therefore r = \frac{4.2}{2} = \frac{21}{10}$

$= 150 + \frac{1386}{100} = 150 + 13.86 = \mathbf{163.86 \text{ cm}^2}$

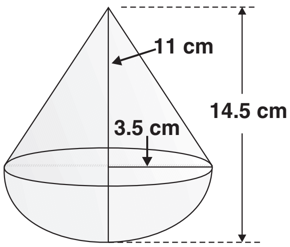
Q. 3. *A toy is in the form of a cone mounted on a hemisphere with same radius. The diameter of the base of the conical portion is 7 cm and the total height of the toy is 14.5 cm. Find the volume of the toy.* [*Use* $\pi = \frac{22}{7}$] (AI CBSE 2007)

Sol. $\begin{bmatrix}\text{Radius of the base of} \\ \text{the conical portion}\end{bmatrix} = \begin{bmatrix}\text{Radius of the} \\ \text{hemisphere}\end{bmatrix}$

$\Rightarrow \quad r = 3.5$ cm,

Height of the conical portion (h)

$$= 14.5 - 3.5 = 11 \text{ cm}$$



Now, Volume of the toy

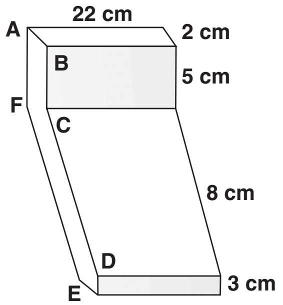
$$= \begin{bmatrix} \text{Volume of the} \\ \text{conical part} \end{bmatrix} + \begin{bmatrix} \text{Volume of the} \\ \text{hemispherical part} \end{bmatrix}$$

$$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^3 [2r + h]$$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \left[2 \times \frac{7}{2} + 11\right] = \frac{11 \times 7}{3 \times 2} \times 18 \text{ cm}^3$$

$$= 11 \times 7 \times 3 \text{ cm}^3 = \mathbf{231 \text{ cm}^3}.$$

Q. 4. *In the figure, the shape of a solid copper piece (made of two pieces) with dimensions as shown. The face ABCDEFA has uniform cross section. Assume that the angles at A, B, C, D, E and F are right angles. Calculate the volume of the piece.* (AI CBSE 2007)



Sol. Since, volume of a cuboid = lbh

$\therefore$ Volume of horizontal piece = $22 \times (8 + 2) \times 3 \text{ cm}^3 = 22 \times 10 \times 3 \text{ cm}^3 = 660 \text{ cm}^3$

Volume of the vertical piece = $22 \times 2 \times 5 \text{ cm}^3 = 220 \text{ cm}^3$

$\therefore$ Total volume of the piece = $660 \text{ cm}^3 + 220 \text{ cm}^3 = \mathbf{880 \text{ cm}^3}$

Q. 5. *A toy is in the form of a cone mounted on a hemisphere of common base radius 7 cm. The total height of the toy is 31 cm. Find the total surface area of the toy. [Use $\pi = \frac{22}{7}$]* (CBSE 2007)

Sol. Height of the cone, $h = 31 - 7 = 24$ cm

Radius of the cone = Radius of the hemisphere

$r = 7$ cm.

Now, Slant height, $l = \sqrt{r^2 + h^2} = \sqrt{7^2 + 24^2} = \sqrt{49 + 576}$ cm $= \sqrt{625}$ cm $= 25$ cm

$\therefore$ Total surface area of the toy

$$= 2\pi r^2 + \pi r l = \pi r [2r + l]$$

$$= \frac{22}{7} \times 7 [2 \times 7 + 25] \text{ cm}^2 = 22 \times [14 + 25] \text{ cm}^2$$

$$= 22 \times 39 \text{ cm}^2 = \mathbf{858 \text{ cm}^2}$$

III. LONG ANSWER TYPE QUESTIONS

Q. 1. *A bucket is in the form of a frustum of a cone. Its depth is 15 cm and the diameters of the top and the bottom are 56 cm and 42 cm respectively. Find how much water can the bucket hold?* [*use* $\pi = \frac{22}{7}$]

Sol. Here $r_1 = \frac{56}{2} = 28$ cm

$r_2 = \frac{42}{2} = 21$ cm

Height $h = 15$ cm

Since the volume of a frustum of a cone $= \frac{1}{3}\pi h\,(r_1^2 + r_2^2 + r_1 + r_2)$

∴ Volume of the bucket

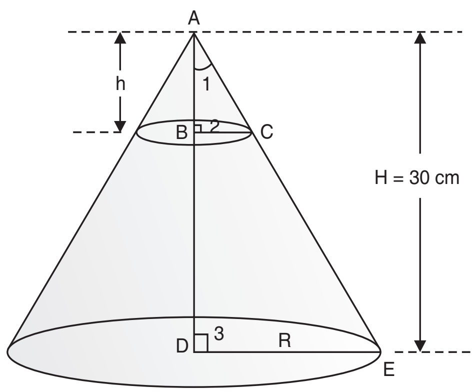
$$= \frac{1}{3} \times \frac{22}{7} \times 15\ [28^2 + 21^2 + 28 \times 21]\ \text{cm}^3$$

$$= \frac{110}{7}[784 + 441 + 588]\ \text{cm}^3 = \frac{110}{7} \times 1813\ \text{cm}^3$$

$$= 110 \times 259\ \text{cm}^3 = 28490\ \text{cm}^3$$

$$= \frac{28490}{1000}\ \text{litres} = \mathbf{28.49\ litres}$$

Q. 2. *The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to the base. If its volume be $\frac{1}{27}$th of the volume of the given cone, at what height above the base is the section made?*



Sol. In the figure, we have

$\Delta ABC \sim \Delta ADE$ [*By AA similarity*]

$$\Rightarrow \frac{BC}{DE} = \frac{AB}{AD}$$

[∵ *corresponding sides of similar* Δ^S *are proportional.*]

$$\Rightarrow \frac{r}{R} = \frac{h}{30} \quad \text{...(1)}$$

Here, volume of the small cone $= \frac{1}{3}\pi r^2 h$

Volume of the given cone $= \frac{1}{3}\pi R^2 H$

Since, [Volume of the small cone] $= \frac{1}{27}$ [Volume of the given cone]

$$\Rightarrow \frac{1}{3}\pi r^2 h = \frac{1}{27}\left(\frac{1}{3}\pi R^2 H\right)$$

$$\Rightarrow \frac{r^2}{R^2} = \frac{\frac{1}{27} \times \frac{1}{3} \times \pi H}{\frac{1}{3} \times \pi \times h} = \frac{1}{27} \times \frac{30}{h} = \frac{10}{9h}$$

But $\qquad \dfrac{r}{R} = \dfrac{h}{30}$, from (1)

$\therefore \qquad \left(\dfrac{h}{30}\right)^2 = \dfrac{10}{9h}$

$\Rightarrow \qquad \dfrac{h^2}{900} = \dfrac{10}{9h}$

$\Rightarrow \qquad 9h^3 = 10 \times 900 = 9000$

$\Rightarrow \qquad h^3 = 1000 = 10^3$

$\Rightarrow \qquad h = 10 \Rightarrow AB = 10$ cm

$\therefore$ The required height $BD = AD - AB = (30 - 10)$ cm = **20 cm**

Q. 3. *A hollow cone is cut by a plane parallel to the base and the upper portion is removed. If the curved surface of the remainder is $\frac{8}{9}$ of the curved surface of the whole cone, find the ratio of the line segments into which the altitude of the cone is divided by the plane.*

Sol. In the figure, we have

$\Delta ABC \sim \Delta ADE$ [*AA similarity*]

$\therefore \qquad \dfrac{BC}{DE} = \dfrac{AB}{AD} = \dfrac{AC}{AE}$ [$\because$ *sides of similar Δ^s are proportional.*]

$\Rightarrow \qquad \dfrac{r}{R} = \dfrac{h}{H} = \dfrac{l}{L}$ ...(1)

Now, curved surface area of the small cone = πrl

Curved surface area of the whole cone = πRL

Since,

[Surface area of the remainder (frustum)]

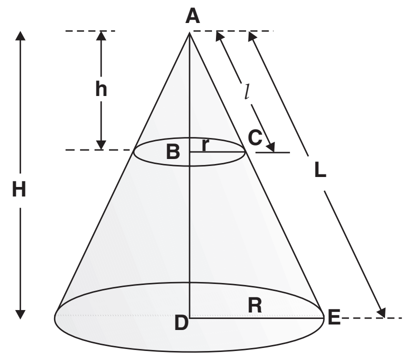
$= \dfrac{8}{9}$ [curved surface area of the given cone]

$\therefore$ [curved surface area of the frustum]

$= \dfrac{8}{9}$ [curved surface area of the given cone]

$\Rightarrow$ [curved surface area of the small cone]

$= \dfrac{1}{9}$ [curved surface area of the whole cone]



$\Rightarrow \qquad \pi rl = \dfrac{1}{9}\pi RL$

$\Rightarrow \qquad \dfrac{\pi rl}{\pi RL} = \dfrac{1}{9}$

$\Rightarrow \qquad \left(\dfrac{r}{R}\right) \times \left(\dfrac{l}{L}\right) = \dfrac{1}{9}$

$\Rightarrow \qquad \left(\dfrac{h}{H}\right) \times \left(\dfrac{h}{H}\right) = \dfrac{1}{9}$

$\Rightarrow \qquad \dfrac{h^2}{H^2} = \left(\dfrac{1}{3}\right)^2$

$$\Rightarrow \quad \frac{h}{H} = \frac{1}{3}$$

$$\Rightarrow \quad H = 3h$$

Now, $BD = H - h = 3h - h = 2h$

$$\therefore \quad \frac{AB}{BD} = \frac{h}{2h} = \frac{1}{2}$$

$\Rightarrow$ **$AB : BD = 1 : 2$**

Q. 4. *A circus tent is cylindrical to a height of 3 m and conical above it. If its base radius is 52.5 m and slant height of the conical portion is 53 m, find the area of the canvas needed to make the tent.* [*Use* $\pi = \frac{22}{7}$].

Sol. For cylindrical part:

We have, radius (r) = 52.5 m

Height (h) = 3 m

Curved surface area = $2\pi rh$

For the conical part

Slant height (l) = 53 m

Radius (r) = 52.5 m

$\therefore$ Curved surface area = πrl

Area of the canvas = $2\pi rh + \pi rl$

$= \pi r(2h + l)$

$$= \frac{22}{7} \times 52.5 \times (2 \times 3 + 53) \text{ m}^2 = \frac{22}{7} \times \frac{525}{10} \times 59 \text{ m}^2$$

$$= \frac{22 \times 75 \times 59}{10} \text{ m}^2 = 11 \times 15 \times 59 \text{ m}^2 = \mathbf{9735 \text{ m}^2}$$

Q. 5. *An iron pillar has some part in the form of a right circular cylinder and remaining in the form of a right circular cone. The radius of the base of each of cone and cylinder is 8 cm. The cylindrical part is 240 cm high and the conical part in 36 cm high. Find the weight of the pillar if one cu. cm of iron weighs 7.8 grams.*

Sol. Here, height of the cylindrical part

h = 240 cm

Height of the conical part, H = 36 cm

Radius r = 8 cm

Now, the total volume of the pillar

$$= \begin{bmatrix} \text{volume of the} \\ \text{cylindrical part} \end{bmatrix} + \begin{bmatrix} \text{volume of the} \\ \text{conical part} \end{bmatrix}$$

$$= [\pi r^2 h] + \left[\frac{1}{3}\pi r^2 H\right] = \pi r^2 \left[rh + \frac{1}{3}H\right]$$

$$= \frac{22}{7} \times 8^2 \left[8 \times 240 + \frac{1}{3} \times 36\right] \text{cm}^3 = \frac{22 \times 64}{7}[240 + 12] \text{ cm}^3$$

$$= \frac{22 \times 64}{7} 252 \text{ cm}^3 = 22 \times 64 \times 36 \text{ cm}^3 = 50688 \text{ cm}^3$$

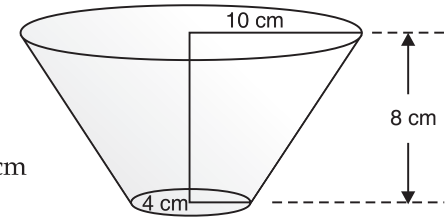
Weight of the pillar $= 7.8 \times 50688 \text{ g} = \frac{78}{10} \times \frac{50688}{1000} \text{ kg}$

$= \frac{3953664}{10000} \text{ kg} = 395.3664 \text{ kg} \approx \mathbf{395.36 \text{ kg.}}$

Q. 6. *An open container made up of a metal sheet is in the form of a frustum of a cone of height 8 cm with radii of its lower and upper ends as 4 cm and 10 cm respectively. Find the cost of oil which can completely fill the container at the rate of ₹ 50 per litre. Also, find the cost of metal used, if it costs ₹ 5 per 100 cm*^{2}. [*Use* $\pi = 3.14$]

Sol. Here, for the frustum,

$r_1 = 10 \text{ cm}, \quad r_2 = 4 \text{ cm} \quad \text{and} \quad h = 8 \text{ cm}$



$\therefore \quad l = \sqrt{h^2 + (r_1 - r_2)^2}$

$= \sqrt{8^2 + (10-4)^2}$

$= \sqrt{64 + 6^2} \text{ cm}$

$= \sqrt{64 + 36} \text{ cm} = \sqrt{100} \text{ cm}$

$= 10 \text{ cm}$

$\therefore$ Volume of the container

$= \frac{1}{3} \pi h \, (r_1^2 + r_2^2 + r_1 r_2)$

$= \frac{1}{3} \times \frac{314}{100} \times 8 \, [10^2 + 4^2 + 10 \times 4] \text{ cm}^3$

$= \frac{314 \times 8}{3 \times 100} \times [100 + 16 + 40] \text{ cm}^3 = \frac{314 \times 8}{3 \times 100} \times 156 \text{ cm}^3$

$= \frac{130624}{100} \text{ cm}^3 = 1306.24 \text{ cm}^3 = \frac{130624}{100 \times 1000} \text{ litres}$

$= 1.30624 \text{ litres} \approx 1.31 \text{ approx.}$

Cost of oil $= ₹\, (1.31 \times 50) = \mathbf{₹\ 65.50}$

Now, the total surface area of the container (excluding upper base)

$= \pi l \, (r_1 + r_2) + \pi r_2^2 = \frac{314}{100} \times 10 \times (10 + 4) + \frac{314}{100} \times 4^2 \text{ cm}^2$

$= \frac{314}{100} [10 \times 14 + 16] \text{ cm}^2 = \frac{314}{100} \times 156 \text{ cm}^2 = \frac{48984}{100} \text{ cm}^2 = 489.84 \text{ cm}^2$

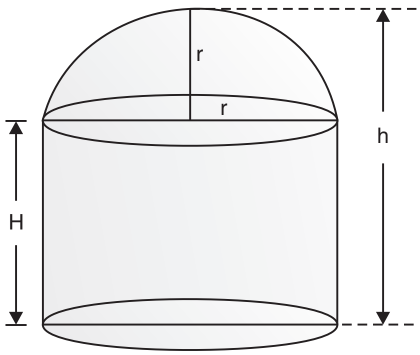
$\therefore$ Cost of metal $= ₹ \frac{5}{100} \times (489.84) = ₹\ 24.492 = \mathbf{₹\ 24.49}$ (appx.)

Q. 7. *A building is in the form of a cylinder surmounted by a hemispherical dome as shown in the figure. The base diameter of the dome is equal to* $\frac{2}{3}$ *of the total height of the building. Find the height of the building, if it contains* $67\frac{1}{21}$ m^3 *of air.* [NCERT Exemplar]

Sol. Here, radius of the hemispherical part = r (say)

Let the total height of the building = h

And the height of the cylindrical part = H



$$\because \left[\begin{array}{c}\text{Diameter of} \\ \text{base of the dome}\end{array}\right] = \frac{2}{3}\left[\begin{array}{c}\text{Total height of} \\ \text{the building}\end{array}\right]$$

$$\therefore \quad 2r = \frac{2}{3}h$$

$$\Rightarrow \quad r = \frac{1}{2}\times\frac{2}{3}h = \frac{h}{3}$$

$$\therefore \quad H = \left(h - \frac{h}{3}\right) \text{ metres} = \frac{2}{3}h \text{ metres}$$

$$\text{Now,} \left[\begin{array}{c}\text{Volume of air} \\ \text{inside the} \\ \text{building}\end{array}\right] = \left[\begin{array}{c}\text{Volume of} \\ \text{air inside the} \\ \text{dome}\end{array}\right] + \left[\begin{array}{c}\text{Volume of air inside the} \\ \text{cylindrical part}\end{array}\right]$$

$$= \frac{2}{3}\pi r^3 + \pi r^2 H$$

$$= \frac{2}{3}\pi\left[\frac{h}{3}\right]^3 + \pi\left[\frac{h}{3}\right]^2 \times \left[\frac{2}{3}h\right]$$

$$= \pi\cdot\frac{8}{81}\cdot h^3 \text{ m}^3$$

But, volume of the air in the building

$$= 67\frac{1}{21} \text{ m}^3$$

$$\therefore \quad \pi \times \frac{8}{81} \times h^3 = 67\frac{1}{21}$$

$$\Rightarrow \quad \frac{22}{7}\times\frac{8}{81}\times h^3 = \frac{1408}{21}$$

$$\Rightarrow \quad h^3 = \frac{1408}{21}\times\frac{7}{22}\times\frac{81}{8}$$

$$\Rightarrow \quad h^3 = 8 \times 27$$

$$= 216$$

$$\Rightarrow \quad h^3 = (6)^3$$

$$\Rightarrow \quad h = 6 \text{ m}$$

Thus, the required height of the building is **6** metres.

Q. 8. *A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 4 cm and diameter of the base is 8 cm. Determine the volume of the toy. If the cube circumscribes the toy, then find the difference of the volumes of the cube and the toy. Also, find the total surface area of the toy.* [NCERT Exemplar]

Sol. Let the radius of the hemisphere = r

And height of the cone = h

Now, $\begin{bmatrix}\text{Volume of} \\ \text{the toy}\end{bmatrix} = \begin{bmatrix}\text{Volume of} \\ \text{the cone}\end{bmatrix} + \begin{bmatrix}\text{Volume of} \\ \text{the hemisphere}\end{bmatrix}$

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \left[\frac{1}{3}\times\frac{22}{7}\times 4^2 \times 4\right] + \left[\frac{2}{3}\times\frac{22}{7}\times 4^3\right] \text{cm}^3$$

$$= \frac{1}{3}\times 4^3 \times \frac{22}{7}[1+2]\,\text{cm}^3 = \frac{1}{3}\times 4\times 4\times 4\times\frac{22}{7}\times 3\ \text{cm}^3$$

$$= \frac{64\times 22}{7}\text{cm}^3 = \mathbf{\frac{1408}{7}\ cm^3}$$

Since, the cube circumscribing the given solid must have its edge as (4 + 4) cm *i.e.,* 8 cm,

$\therefore$ Volume of the cube = $(\text{edge})^3 = (8)^3\ \text{cm}^3 = \mathbf{512\ cm^3}$

Now, difference in volumes of the cube and the toy

$$= \left[512 - \frac{1408}{2}\right]\text{cm}^3$$

$$= \frac{3584 - 1408}{7}\text{cm}^3$$

$$= \frac{2176}{7}\text{cm}^3 = \mathbf{310.86\ cm^3}$$

Also, total surface area of the toy

$$= \begin{bmatrix}\text{curved surface area} \\ \text{of conical part}\end{bmatrix} + \begin{bmatrix}\text{curved surface area} \\ \text{of the hemispherical part}\end{bmatrix}$$

$$= \pi r l + 2\pi r^2 \qquad \left[\textit{where, } l = \sqrt{h^2 + r^2}\ \right]$$

$$= \pi r\,[l + 2r] = \frac{22}{7}\times 4\times\left[(\sqrt{4^2+4^2}) + 2\times 4\right]\text{cm}^2$$

$$= \frac{22}{7}\times 4\times[4\sqrt{2} + 8]\,\text{cm}^2$$

$$= \frac{22}{7}\times 4\times 4\,[\sqrt{2} + 2]\,\text{cm}^2 = \frac{352}{7}(\sqrt{2} + 2) = \mathbf{171.68\ cm^2}$$

TEST YOUR SKILLS

1. The diameter of a sphere is 28 cm. Find the cost of painting it all around at ₹ 0.10 per square cm.
2. The perimeter of one face of a wooden cube is 20 cm. Find its weight if 1 cm^3 of wood weighs 8.25 g.
3. The radii of two cylinders are in the ratio of $1 : \sqrt{3}$. If the volumes of two cylinders be same, find the ratio of their respective heights.
4. If the radius of the base of a cone is doubled keeping the height same. What is the ratio of the volume of the larger cone to the smaller cone?
5. If the length, breadth and height of a solid cube are in the ratio 4 : 3 : 2 and total surface area is 832 cm^2. Find its volume.
6. Three cubes of a metal whose edges are in the ratio 3 : 4 : 5 are melted and converted into a single cube whose diagonal is $12\sqrt{3}$ cm. Find the edges of the three cubes.

[NCERT Exemplar]

Hint:

Let the edges of the given cubes be 3x, 4x and 5x.

$\therefore$ *Sum of volumes of the given cubes* $= (3x)^3 + (4x)^3 + (5x)^3 = 216x^3$

Let edge of the new cube be 'a'.

$\therefore$ *Diagonal of the new cube* $= \sqrt{a^2 + a^2 + a^2} = \sqrt{3a^2} = a\sqrt{3}$

$\Rightarrow a\sqrt{3} = 12\sqrt{3}$ *or* $a = 12$

Also, volume of the new cube $= a^3 = 216\ x^3$

$\Rightarrow 12^3 = 216\ x^3 \Rightarrow x = 2$

$\therefore 3x = 6,\ 4x = 8$ *and* $5x = 10$

7. A toy is in the form of a cone mounted on a hemisphere of common base of diameter 7 cm. If the height of the toy is 15.5 cm, find the total surface area of the toy. [Take $\pi = \frac{22}{7}$]

[CBSE 2012]

8. A circular tent is cylindrical up to a height of 3 m and conical above it. If the diameter of the base of the cone and cylinder is 105 m and the slant height of the conical part is 53 m, find the total canvas used in making the tent. [CBSE 2012]
9. A solid composed of a cylinder with hemispherical ends. If the whole length of the solid is 108 cm and the diameter of the hemispherical end is 36 cm, find the cost of polishing its surface at the rate of 70 paise per square cm. [CBSE 2012]
10. A 5 m wide cloth is used to make a conical tent of base diameter 14m and height 24m. Find the cost of cloth used at the rate of ₹ 25 per metre [Use $\pi = \frac{22}{7}$]

[AI. CBSE (Foreign) 2014]

Hint:

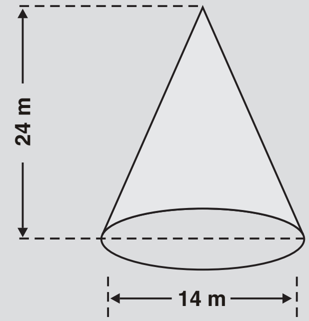
Slant height $= \sqrt{h^2 + r^2} = \sqrt{24^2 + 7^2} = 25\ m$

$\therefore$ *Curved S.A* $= \pi rl = \dfrac{22}{7} \times 25 \times 7\ m^2 = 550\ m^2$

Area of the cloth required = length × breadth

$= l \times 5 = 550 \Rightarrow l = \dfrac{550}{5} = 110\ m$

Cost of the cloth = ₹ 25 × 110 = ₹ 2750 *per metre.*



11. A girl empties a cylindrical bucket, full of sand, of base radius 18 cm and height 32 cm, on the floor to form a conical heap of sand. If the height of this conical heap is 24 cm, then find its slant height correct up to one place of decimal. [AI. CBSE (Foreign) 2014]

Hint:

Let radius and height of cylindrical bucket = R and H respectively

Let r and 'h' be radius and height of conical heap.

$$\therefore \quad \pi R^2 H = \frac{1}{3}\pi r^2 h \Rightarrow r = \sqrt{\frac{3 \times H \times R^2}{h}}$$

$$= \sqrt{\frac{3 \times 18 \times 18 \times 32}{24}} = 36$$

$$\therefore \quad l = \sqrt{r^2 + h^2} = \sqrt{36^2 + 24^2}$$

$$= \sqrt{1296 + 574}$$

$$= \sqrt{1872} = 43.3\,cm$$

12. The largest possible sphere is carved out of a wooden solid cube of side 7 cm. Find the volume of the wood left. [Use $\pi = \dfrac{22}{7}$] [AI. CBSE 2014]

Hint:

Volume of the wooden cube $= 7 \times 7 \times 7 cm^3 = 343 cm^3$

Radius of the sphere $= \dfrac{1}{2}$ *side of the cube* $= \dfrac{1}{2}(7\ cm) = 3.5\ cm$

$$\therefore \quad \text{Volume of the sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^3 cm^3 = 179.66\ cm^3$$

$$\Rightarrow \quad \text{Volume of the wood left} = \begin{bmatrix}\text{volume of} \\ \text{cube}\end{bmatrix} - \begin{bmatrix}\text{volume} \\ \text{of sphere}\end{bmatrix} = (343 - 179.66)\,cm^3$$

$$= 163.34\ cm^3$$

13. 150 spherical marbles, each of radius 1.4 cm, are dropped in a cylindrical vessel of radius 7 cm containing some water, which are completely immersed in water. Find the rise in the level of water in the vassel. [AI. CBSE 2014]

Hint:

Volume of 150 spherical marbles = 150 × *[volume of a spherical marble]*

$$= 150 \times \frac{4}{3} \times \frac{22}{7} \times (1.4)^3 = 548.8\,cm^3$$

$\therefore$ *Let the level of water rises h cm.*

$\therefore$ *Volume of 'h' high water in the vessel* = $\pi \times r^2 \times h\,cm^3 = \frac{22}{7} \times 7 \times 7 \times h = 154\,h\,cm^3$

$\Rightarrow$ $154\,h = 548.8 \Rightarrow h = \frac{5488}{10} \times \frac{1}{154} = 3.56$ *cm*

ANSWERS

Test Your Skills

1. ₹ 246.40 **2.** 1031.25 g **3.** 3 : 1 **4.** 4 : 1 **5.** 1536 cm^3

6. 6 cm, 8 cm, 10 cm **7.** 214.5 cm^2 **8.** 9735 m^2 **9.** ₹ 8553.60

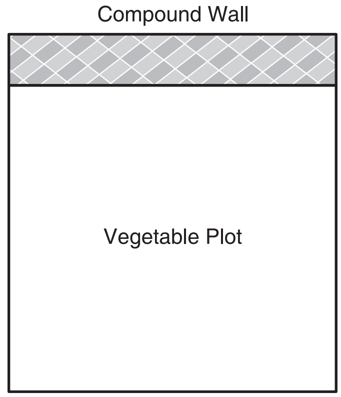
Value Based Questions (Solved)

Q1. *Rahul plans to grow organic vegetables in a 100 sq. m rectangular plot. He has only 30 m of barbed wire which can fence its three sides. Fourth side of his plot touches Rehaman's compound wall. He requests Rehaman to allow his compound wall be used as fencing to his plot.*

(*a*) *Find the dimensions of the plot.*

(*b*) *Which mathematical concept is used in above problem?*

(*c*) *By allowing the compound wall to act as fencing, which value is depicted by Rehaman?*



Sol. Let 'x' metres be the width and 'y' metres be the length of the vegetable plot.

$\therefore$ Area $= xy$ sq. m

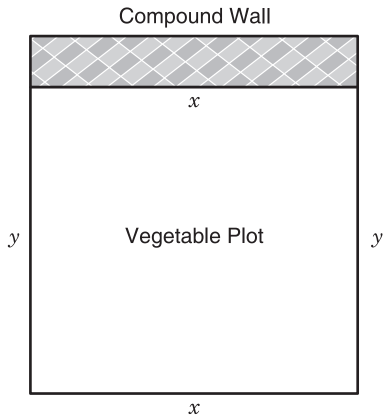
$\Rightarrow$ $xy = 100$ ... (*i*)

$\because$ length of the barbed wire $= 30$ m

$x + y + y = 30$ m

$\Rightarrow$ $x + 2y = 30$

$\Rightarrow$ $y = \dfrac{30 - x}{2}$ ... (*ii*)



From (*i*) and (*ii*), we have

$$x\left(\frac{30 - x}{2}\right) = 100$$

$\Rightarrow$ $x^2 - 30x + 200 = 0$

$\Rightarrow$ $(x - 10)(x - 20) = 0$

$\Rightarrow$ $x = 10$ or $x = 20$

For $x = 10$, we get $y = \dfrac{100}{x} = \dfrac{100}{10} = 10$

For $x = 20$, we get $y = \dfrac{100}{x} = \dfrac{100}{20} = 5$

Thus,

(*a*) The dimensions of the plot are: 10m, 10m or 20m, 5m.

(*b*) Quadratic Equations

(*c*) Co-operation.

Q2. *A shopkeeper buys a certain number of books from a publisher for ₹ 80. The publisher gives him 4 more books for same amount with a condition that the shopkeeper would donate ₹ 1 per book to an orphanage.*

(*a*) *How many books did he buy?*

(*b*) *Which mathematical concept is used in this problem?*

(*c*) *By allowing ₹ 1 per book towards an orphanage, which value is depicted by the publisher?*

Sol. Let the number of books bought = x

$\therefore$ Cost of x books = ₹ 80

$\Rightarrow$ Cost of 1 book = ₹ $\frac{80}{x}$

Again,

Cost of $(x + 4)$ books = ₹ 80

$\Rightarrow$ Cost of 1 book = ₹ $\frac{80}{(x+4)}$

Since, the shopkeeper donates ₹ 1 per book to an orphanage.

$\therefore$ $\frac{80}{x} - \frac{80}{(x+4)} = 1$ $\Rightarrow$ $80\left[\frac{1}{x} - \frac{1}{x+4}\right] = 1$

$\Rightarrow$ $80\left[\frac{x+4-x}{x^2+4x}\right] = 1$ $\Rightarrow$ $\frac{320}{x^2+4x} = 1$

$\Rightarrow$ $x^2 + 4x = 320$ $\Rightarrow$ $x^2 + 4x - 320 = 0$

$\Rightarrow$ $x^2 + 20x - 16x - 320 = 0$ $\Rightarrow$ $x(x + 20) - 16(x + 20) = 0$

$\Rightarrow$ $(x + 20)(x - 16) = 0$ $\Rightarrow$ $x = -20$ or $x = 16$

$\because$ x cannot of negative

$\therefore$ $x = 16$

Thus,

(*a*) Number of books bought = 16 (*b*) Quadratic Equations (*c*) Charity

Q3. *Radha wants to buy a piece of cloth for ₹ 200. She bought another 5m piece of cloth for donation to a blind-school. For this, the shopkeeper reduces the cost by ₹ 2 per metre such that the total cost remains the same (₹ 200).*

(*a*) *What is the original rate per metre?*

(*b*) *Which mathematical concept is used in the above problem?*

(*c*) *By donating a piece of cloth to the blind-school, which value is depicted?*

Sol. Let the original length of cloth = 'x' metres

$\therefore$ Original rate = ₹ $\frac{200}{x}$

New length of the cloth = $(x + 5)$ metres

New length of cloth = ₹ $\frac{200}{(x+5)}$

$\because$ The new rate is ₹ 2 less than the original rate

$\therefore$ $\frac{200}{x} - \frac{200}{(x+5)} = 2$ $\Rightarrow$ $200\left[\frac{1}{x} - \frac{1}{(x+5)}\right] = 2$

$\Rightarrow$ $200\left[\frac{x+5-x}{x^2+5x}\right] = 2$ $\Rightarrow$ $1000 = 2(x^2 + 5x)$

$\Rightarrow$ $1000 = 2x^2 + 10x$

$\Rightarrow$ $x^2 + 5x - 500 = 0$ $\Rightarrow$ $x^2 + 25x - 20x - 500 = 0$

$\Rightarrow$ $(x + 25)(x - 20) = 0$ $\Rightarrow$ $x = -25$, which is not desirable or $x = 20$

Now,

(*a*) Original rate = ₹ $\frac{200}{20}$ per metre = ₹ 10 per metre

(*b*) Quadratic Equations

(*c*) Charity

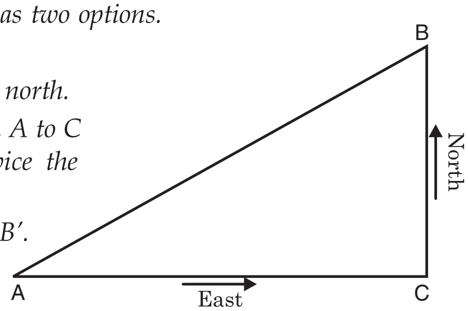
Q4. *Ranjeet wants to go by car from place – 'A' to 'B'. He has two options.*

(i) He can go straight from A to B.

(ii) He goes to 'C' due east and then from 'C' to 'B' due north.

The distance between A to B exceeds the distance between A to C by 2 km. The distance between 'A' to 'B' exceeds twice the distance between 'C' and 'B' by 1 km.

He decided to choose the option (i) for going from 'A' to 'B'.



(a) Find the distance difference in the above two options.

(b) Which mathematical concept is used in the above problem?

(c) By choosing the option (i), which value is depicted by Ranjeet?

Sol. Let the distance between 'C' and 'B' = x km

$\therefore$ Distance between 'A' and 'B' = $(2x + 1)$ km

And distance between 'A' and 'C' = $(2x + 1) - 2$ km

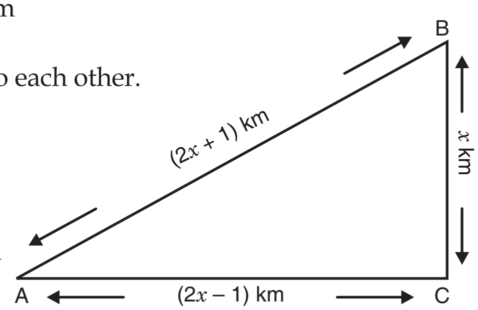
$= (2x - 1)$ km

$\because$ The direction East and North are perpendicular to each other.

$\therefore$ AC $\perp$ BC

$\Rightarrow$ ABC is a right angle Δ, right angled at C

$\therefore$ Using Pythagoras theorem, we have:



$\Rightarrow \quad x^2 + (2x - 1)^2 = (2x + 1)^2$

$\Rightarrow \quad x^2 + 4x^2 - 4x + 1 = 4x^2 + 4x + 1$

$\Rightarrow \quad x^2 - 8x = 0$

$\Rightarrow \quad x(x - 8) = 0$

$\Rightarrow$ Either $x = 0$, [Which is not possible as distance cannot be zero]

Or $x = 8$

Now, AB = $(2x + 1)$ km = $(16 + 1)$ km = 17 km

AC = $(2x - 1)$ km = $(16 - 1)$ km = 15 km

BC = x km = 8 km

Now, (*a*) Difference in distance = (AC + BC) – AB = (15 + 8) km – 17 km

= 23 km – 17 km = 6 km

(*b*) Quadratic Equations

(*c*) Saving of National resource (fuel consumption)

Q5. *Savita has two options to buy a house:*

(a) She can pay a lumpsum amount of ₹ 22,00,000

Or

(b) She can pay ₹ 4,00,000 cash and balance in 18 annual instalments of ₹ 1,00,000 plus 10% interest on the unpaid amount.

She prefers the option (i) and donates 50% of the difference of the costs in the above two options to Prime Minister Relief Fund.

(*i*) *What amount was donated to Prime Minister Relief Fund?*

(*ii*) *Which mathematical concept is used in the above problem?*

(*iii*) *By choosing to pay a lumpsum amount and donating 50% of the difference to Prime Minister Relief Fund, which value is depicted by Savita?*

Sol. (*a*) Total cost of the house = ₹ 22,00,000

(*b*) Cash payment = ₹ 4,00,000

Balance = ₹ 22,00,000 – ₹ 4,00,000 = ₹ 18,00,000

1st instalment = ₹ [1,00,000 + 10% of balance]

$= ₹ \left[1,00,000 + \frac{10}{100} \times 18,00,000\right]$

= ₹ [1,00,000 + 1,80,000] = ₹ 2,80,000

Balance after 1st instalment = ₹ [18,00,000 – 1,00,000] = ₹ 17,00,000

2nd instalment = ₹ [1,00,000 + 10% of 17,00,000]

= ₹ [1,00,000 + 1,70,000] = ₹ [2,70,000]

Balance after 2nd instalment = ₹ 17,00,000 – ₹ 1,00,000 = ₹ 16,00,000

∴ 3rd instalment = ₹ [1,00,000 + 10% of 16,00,000]

= ₹ [1,00,000 + 1,60,000] = ₹ 2,60,000

... and so on.

∴ Total amount in instalments = ₹ 2,80,000 + ₹ 2,70,000 + ₹ 2,60,000 + ... to 18 terms

$= \frac{n}{2}[2a+(n-1)d]$, where a = 2,80,000, d = –10,000, n = 18

$= ₹ \frac{18}{2}[2(2,80,000) + (18 - 1)(-10,000)$

$= ₹\, 9\,[5,60,000 + 17(-10,000)]$

$= ₹\, 9\,[5,60,000 - 1,70,000]$

$= ₹\, 9\,[3,90,000]$ = ₹ 35,10,000

∴ Total cost of house = ₹ 35,10,000 + 4,00,000 = ₹ 39,10,000

Difference in costs of the house in two options

= ₹ 39,10,000 – ₹ 22,00,000 = ₹ 17,10,000

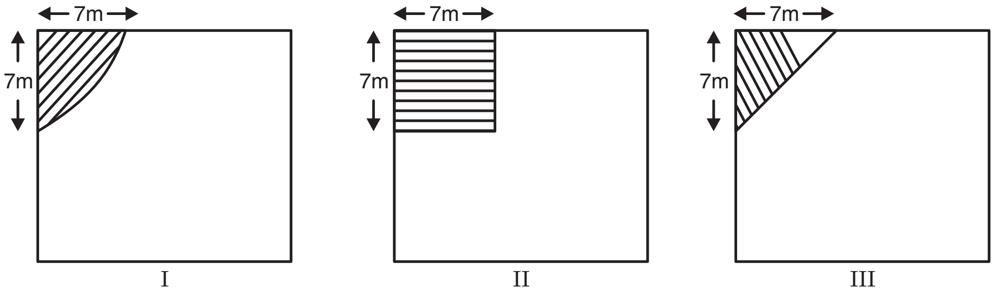
∴ (*i*) Amount donated towards Prime Minister Relief Fund = 50% of ₹ 17,10,000

$= ₹ \frac{50}{100} \times 17,10,000$ = ₹ 8,55,000

(*ii*) Arithmetic Progressions

(*iii*) National Loyality

Q6. *Shankar wants a lawn to be developed in a corner of his square plot. He gave following three options to the lawn-developer:*



The lawn developing rate is ₹ 150 per sq. m and the lawn under option-III was developed, however, Shankar paid for the option having largest area. The lawn-developer realized the mistake and refunded the balance back to Shankar.

(a) *Find the area in each of the above three options.*

(b) *What amount was refunded back to Shankar by the lawn-developer?*

(c) *Which mathematical concept is used in above problem?*

(d) *Refunding the difference, which value is depicted by the lawn-developer?*

Sol. (a) Area of lawn in option-I $= \frac{1}{4}(\pi r^2)$

$= \frac{1}{4} \times \frac{22}{7} \times 7 \times 7 \text{ m}^2 = \frac{154}{4} \text{ m}^2 = 38.5 \text{ m}^2$

Area of lawn in option-II $= 7 \times 7 \text{ m}^2 = 49 \text{ m}^2$

Area of lawn in option-III $= \frac{1}{2} \times 7 \times 7 \text{ m}^2 = \frac{49}{2} \text{ m}^2 = 24.5 \text{ m}^2$

(b) $\because$ Largest area of the lawn $= 49 \text{ m}^2$

Cost of lawn-developing in option-II

$= ₹\ 150 \times 49 = ₹\ 7350$

$\because$ Area of lawn in option-III $= 24.5 \text{ m}^2$

$\therefore$ Cost of lawn-developing in option-III

$= ₹\ 150 \times 24.5 = ₹\ 150 \times \frac{245}{10}$

$= ₹\ 15 \times 245 = ₹\ 3675$

$\Rightarrow$ Amount refunded back by the lawn-developer

$= ₹\ 7350 - ₹\ 3675 = ₹\ 3675$

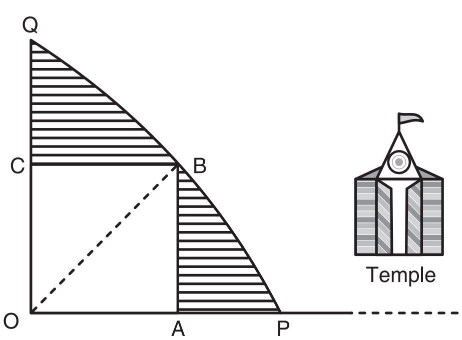
(c) Areas related to plane surfaces.

(d) Honesty

Q7. *Shivram has a piece of land in the form of sector OBPQ adjoining to a temple. He donates a part of it to the temple such that a square plot OABC is left with him. If OA = 20 m. Then,*

(a) *Find the area of the shaded region (donated to the temple). [use $\pi = 3.14$]*

(b) *Which mathematical concept is used in above problem?*



(c) *By donating land to a temple by Shivram, which value is depicted?*

Sol. (a) $\because$ OABC is a square and OA = 20 m

$\therefore$ Diagonal OB of the square OABC

$$= \sqrt{20^2 + 20^2} = \sqrt{400 + 400} = \sqrt{800}\text{ m}$$

$\Rightarrow$ Radius of the quadrant OPBQ = $\sqrt{800}$ m

$\Rightarrow$ Area of the quadrant OPBQ = $\frac{1}{4}\pi r^2 = \frac{1}{4} \times \frac{314}{100} \times \left[\sqrt{800}\right]^2 \text{ m}^2$

$$= \frac{1}{4} \times \frac{314}{100} \times 800 \text{ m}^2 = \frac{1}{4} \times 314 \times 8 \text{ m}^2$$

$$= 314 \times 2 \text{ m}^2 = 628 \text{ m}^2$$

Also, area of square OABC = 20×20 m^2 = 400 m^2

$\therefore$ Area of the shaded region = [Area of the quadrant OPBQ] – [Area of the square OABC]

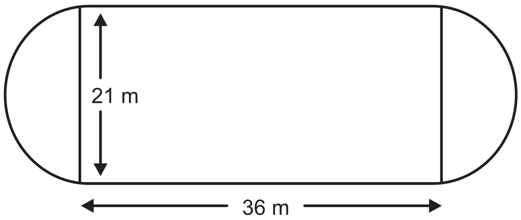
$$= 628 \text{ m}^2 - 400 \text{ m}^2 = 228 \text{ m}^2$$

Thus, area of the land donated to the temple = 228 m^2

(b) Areas related to circles

(c) Charity.

Q8. *Rajat has a piece of land in the shape of a rectangle with two semicircles on its smaller sides, as diameters, added to its outside. The sides of the rectangle are 36 m and 21 m. He lets out the two semicircular parts at the rate of ₹ 100 per sq. metre and donates the proceeds to an orphanage.*



(a) *Find the total area of his land. [use* $\pi = \frac{22}{7}$*]*

(b) *What amount does he donate to orphanage?*

(c) *Which mathematical concept is used in the above problem?*

(d) *By donating the rent proceeds to an orphanage which value is depicted by Rajat?*

Sol. (a) Area of the rectangular part = 36×24.5 m^2

$$= 36 \times \frac{245}{10} \text{ m}^2 = 18 \times 49 \text{ m}^2 = 882 \text{ m}^2$$

Area of both the semicircular parts = $2\left[\frac{1}{2}\pi r^2\right]$

$$= 2\left[\frac{1}{2} \times \frac{22}{7} \times 21 \times 21\right] \text{m}^2 = 22 \times 3 \times 21 \text{ m}^2 = 1386 \text{ m}^2$$

$\therefore$ Area of Rajat's total land = 882 m^2 + 1386 m^2 = 2268 m^2

(*b*) ∴ Rent rate = ₹ 100 per m^2

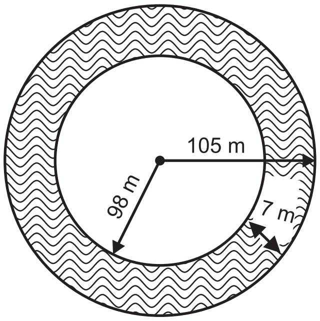
∴ Total rent = ₹ 1386 × 100 = ₹ 1,38,600

⇒ Amount donated to orphanage by Rajat = ₹ 1,38,600

(*c*) Surface areas related to circles

(*d*) Charity

Q9. *Rajesh has a circular plot of radius 105m. He donates a 7m wide track along its boundary for community-track.*



(*a*) *Find the area of the track. [use* $\pi = \frac{22}{7}$*]*

(*b*) *Which mathematical concept is used in the above problem?*

(*c*) *By donating a community-track, which value is depicted by Rajesh?*

Sol. (*a*) Radius of the total circular plot = 105 m

∴ Area of the total circular plot = $\pi (105)^2$ sq. m

∵ Width of the track = 7 m

∴ Radius of the circular plot excluding the track

= 105 m – 7 m = 98 m

⇒ Area of the inner circular plot = $\pi (98)^2$ sq. m.

Now, area of the track = [Area of total circular plot] – [Area of inner circular plot]

$= \pi (105)^2 - \pi (98)^2$ sq. m

$= \pi [105^2 - 98^2]$ sq. m

$= \pi [(105 - 98)(105 + 98)]$ sq. m

[using $a^2 - b^2 = (a-b)(a+b)$]

$= \pi [7 \times 203]$ sq. m

$= \frac{22}{7} \times 7 \times 203$ sq. m

$= 22 \times 203$ sq. m

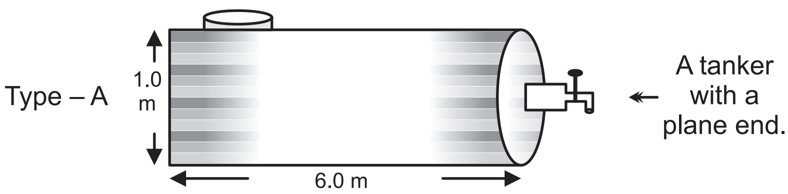
$= 4466$ sq. m

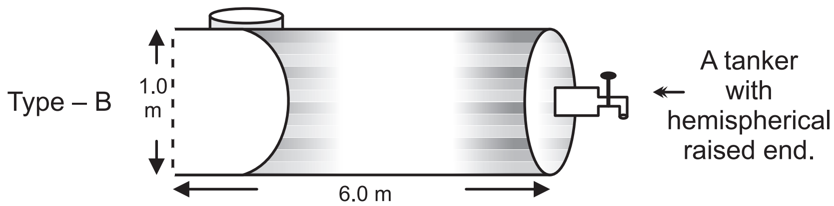
(*b*) Areas related to circles

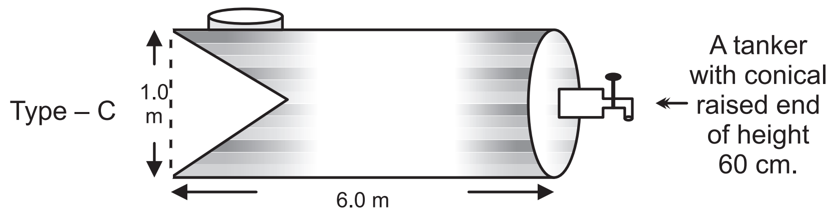
(*c*) Community service

Q10. *Rohan is in the business of supplying water. He has three types of tankers of inner diameter 1.0 m to supply water to the customers. The length of the tankers is 6 m. [use* $\pi = 3.14$*]*

He decided to serve his customers with type 'A' tankers.







(*a*) *Find the volume of tanker of type A.*

(*b*) *Which tanker has the minimum capacity?*

(*c*) *Which mathematical concept is used in the above problem?*

(*d*) *By choosing a tanker of 'type-A', which value is depicted by Rohan?*

Sol. $\therefore$ Diameter = 1.0 m

$\therefore$ Radius $= \frac{1}{2}$ m

Length of the tank = 6 m $\Rightarrow h = 6$ m

(*a*) Volume of the tanker of type-A $= \pi r^2 h$

$$= 3.14 \times \frac{1}{2} \times \frac{1}{2} \times 6 \text{ cu. m}$$

$$= \frac{314}{100} \times \frac{1}{2} \times \frac{1}{2} \times 6 \text{ cu.m} = \frac{471}{100} \text{ cu. m} = 4.71 \text{ cu. m}$$

(*b*) Volume of the tanker of 'type-B'=[Volume of the tanker-A] – [Volume of hemisphere]

$$= 4.71 \text{ cu. m} - \left[\frac{2}{3}\pi r^3\right]$$

$$= 4.71 \text{ cu. m} - \left[\frac{2}{3} \times \frac{314}{100} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right] \text{cu.m}$$

$$= 4.71 \text{ cu. m} - \left[\frac{157}{600}\right] \text{ cu. m}$$

$$= \frac{471}{100} \text{ cu. m} - \frac{157}{600} \text{ cu. m}$$

$$= \frac{2669}{600} \text{ cu. m} = 4.45 \text{ cu. m}$$

Volume of the tanker of 'type-C' = [Volume of the tanker-A] – [Volume of the cone]

$$= 4.71 \text{ cu. m} - \left[\frac{1}{3}\pi r^2 h\right]$$

$$= 4.71 \text{ cu. m} - \left[\frac{1}{3}\times 3.14\times\frac{1}{2}\times\frac{1}{2}\times 0.6\right]\text{cu.m}$$

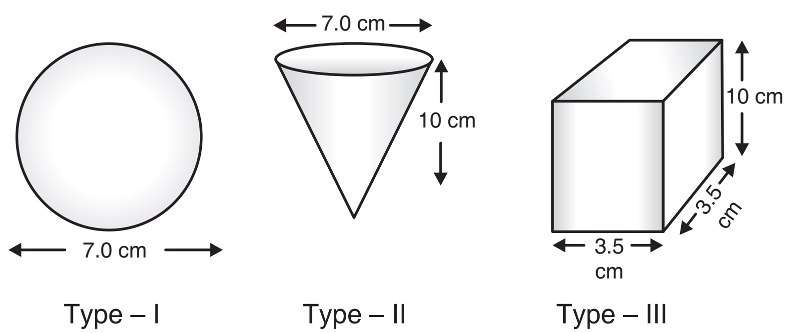
$= [4.71 - 0.157]$ cu. m $= [0.157]$ cu. m $= 4.553$ cu. m

Thus, the tanker of 'type-B' has the minimum capacity of 4.45 cu. m.

(*c*) Mensuration [volume of solid figures]

(*d*) Honesty

Q11. *Shankar, a 'Kulfi-vendor' has three types (spherical, conical and cuboidal) of containers for making Kulfi :*



He decided to serve the customers with 'type-I' container.

(*a*) *Find the volume of the container 'type-I'.*

(*b*) *Which container has the minimum capacity?*

(*c*) *Which mathematical concept is used in this problem?*

(*d*) *By choosing to prepare to sell Kulfi using container of 'type-I', which value is depicted by Shankar?*

Sol. (*a*) Finding the volume of Type-I container:

$\because$ Diameter = 7 cm

$\therefore$ Radius $= \frac{7}{2}$ cm

$\Rightarrow$ Volume of the container of 'type-I' $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times\frac{7}{2}\text{ cu.cm}$$

$$= \frac{11\times 7\times 7}{3}\text{ cu. cm} = 179.66 \text{ cu. cm}$$

(*b*) Volume of conical (type-II) container $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\times\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\times 10\text{ cu.cm}$$

$$= \frac{11 \times 35}{3} \text{ cu.cm} \quad = 128.33 \text{ cu. cm}$$

Volume of cuboidal (type-III) container = $l \times b \times h$

$$= 3.5 \times 3.5 \times 10 \text{ cu. cm} = \frac{35 \times 35}{10} \text{ cu.cm} \quad = 122.5 \text{ cu. cm}$$

Obviously, the capacity of the cuboidal (type-III) container is minimum, which is 122.5 cu. cm.

(*c*) Volume of solid bodies (mensuration)

(*d*) Honesty

Q12. *Prashant has undertaken a contract to build a wall of 9m long, 2.5m thick and 6m high. His labour is to be calculated according to the number of bricks used to complete the wall. In the market three types of bricks are available.*

Type-I : Each measuring 25cm × 11.25cm × 6cm

Type-II : Each measuring 20cm × 8cm × 10cm

Type-III : Each measuring 25cm × 10cm × 9cm

Prashant used bricks of type-III.

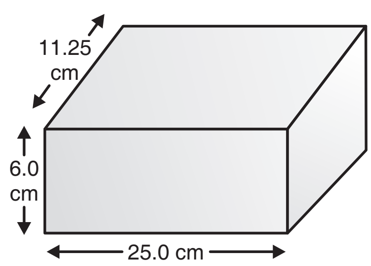
(*a*) *Find the number of bricks of type-III required to build the wall.*

(*b*) *In which case, maximum number of bricks will be used?*

(*c*) *Which mathematical concept is used in the above problem?*

(*d*) *By using the bricks of type-III, which value is depicted by Prashant?*

Sol. Volume of a brick of type-I = $l \times b \times h$



$= 25 \text{ cm} \times 11.25 \text{ cm} \times 6 \text{ cm}$

$= 1687.50 \text{ cm}^3$

Volume of a brick of type-II = $l \times b \times h$

$= 20 \text{ cm} \times 8 \text{ cm} \times 10 \text{ cm} \quad = 1600 \text{ cm}^3$

Volume of a brick of type-III = $l \times b \times h$

$= 25 \text{ cm} \times 10 \text{ cm} \times 9 \text{ cm} \quad = 2250 \text{ cm}^3$

Volume of the wall = $l \times b \times h$

$= 9 \times 2.5 \times 6 \text{ m}^3 \quad = 135 \times 10^6 \text{ cm}^3$

(*a*) ∴ Number of bricks of type-III required for building the wall

$$= \frac{\text{Volume of the wall}}{\text{Volume of a brick of type-III}}$$

$$= \frac{135 \times 10^6}{2250} = 60,000$$

(*b*) Number of bricks required for building the wall using type-I bricks:

$$= \frac{\text{Volume of the wall}}{\text{Volume of a brick of type-I}}$$

$$= \frac{135 \times 10,00,000}{1687.50} = 80,000$$

Number of bricks of type-II required for building the wall

$$= \frac{\text{Volume of the wall}}{\text{Volume of a brick of type-II}}$$

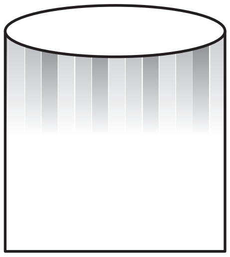
$$= \frac{135 \times 10,00,000}{1600} = 84,375$$

$\Rightarrow$ In case of type-II, the maximum number of bricks will be required.

(*c*) Volume of solid bodies (mensuration)

(*d*) Honesty

Q13. *Sampat has set up his juice shop. He has three types of cylindrical glasses as given below :*

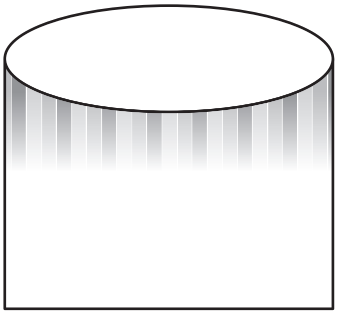


Type – I

A cylindrical glass with inner diameter 7 cm and height as 10 cm.

Type – II

A cylindrical glass with inner diameter 4 cm and height as 14 cm.



Type – III

A cylindrical glass with inner diameter 14 cm and height as 4 cm.

He decided to serve the customers in 'type-I' of glasses.

(*a*) *Find the volume of the glass of type-I.*

(*b*) *Which glass has the minimum capacity?*

(*c*) *Which mathematical concept is used in above problem?*

(*d*) *By choosing a glass of type-I, which value is depicted by juice seller Sampat?*

Sol. (*a*) Diameter of glass of type-I = 7 cm

$\therefore$ Radius = $\frac{7}{2}$ cm

Height = 10 cm

$\Rightarrow$ Volume = $\pi r^2 h$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 10 \text{ cm}^3$

$= 385 \text{ cm}^3$

(*b*) Diameter of glass of type-II=4 cm

$\therefore$ Radius = 2cm

Height = 14 cm

$\Rightarrow$ Volume = $\pi r^2 h$

$= \frac{22}{7} \times 2 \times 2 \times 14 \text{ cm}^3$

$= 176 \text{ cm}^3$

Diameter of glass of type-III = 14 cm

$\therefore$ Radius = $\frac{14}{2}$ cm = 7 cm

Height = 4 cm

$\Rightarrow$ Volume = $\pi r^2 h$

$= \frac{22}{7} \times 7 \times 7 \times 4 \text{ cm}^3$

$= 22 \times 28 \text{ cm}^3$

$= 308 \text{ cm}^3$

$\Rightarrow$ The glass of type-II has the minimum capacity.

(*c*) Mensuration [volume of solid bodies]

(*d*) Honesty.

Q14. *A community-well with 10 m inside diameter is dug 14 m deep. For spreading the earth taken out of it, there are two options :*

(*i*) *It is spread all around the well to a width of 6 m to form an embankment.*

(*ii*) *It is spread evenly on a rectangular surface of 25m × 11m.*

The contractor charges according to the height of the spreadover mud. He charges by choosing the option-(i).

(*a*) *Find the height of the spreadover mud in both options.*

(*b*) *Which mathematical concept is involved in the above problem?*

(*c*) *In which case cost of digging the well is less?*

(*d*) *Which values are depicted by the contractor by charging according to option-(i)?*

Sol. Inside diameter of the well = 10 m

$\Rightarrow$ Inside radius of the well = 5 m

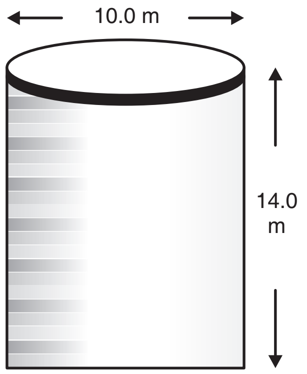
Depth of the well = 14 m

$\therefore$ Volume of the earth dugout = $\pi r^2 h$

$= \frac{22}{7} \times 5 \times 5 \times 14 \text{ m}^3$

$= 22 \times 5 \times 5 \times 2 \text{ m}^3$

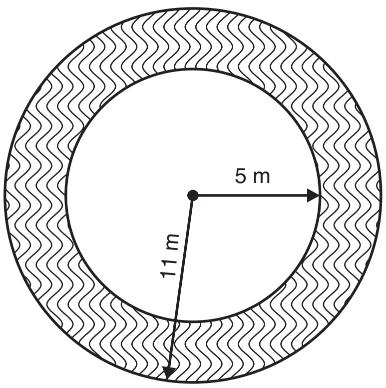
$= 1100 \text{ m}^3$



(*a*) Height of the spreadover mud:

option-(*i*)

Area of the shaded-region (base of the spreadover mud)



$= \pi R^2 - \pi r^2$

$= \pi [R^2 - r^2]$ [Here: R = 11 m and r = 5 m]

$= \pi [(R - r)(R + r)]$

$= \frac{22}{7} [(11 - 5)(11 + 5)] \text{ m}^2$

$= \frac{22}{7} \times 6 \times 16 \text{ m}^2$

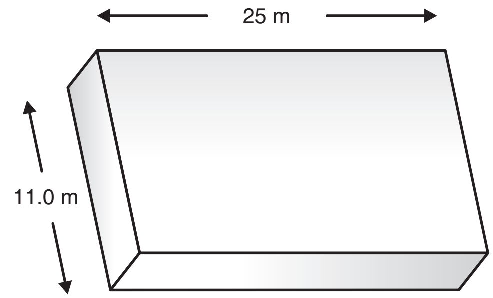
$= \frac{22 \times 96}{7} \text{ m}^2$

Let height of the spreadover mud (embankment) be h_1,

$\therefore \quad \frac{22 \times 96}{7} \times h_1 = 1100$

$\Rightarrow \quad h_1 = \frac{1100 \times 7}{22 \times 96}$

$= 3.64$ m



option-(*ii*)

Let the height of the spreadover mud $= h_2$,

$\therefore \quad 25 \times 11 \times h_2 = 1100$

$\Rightarrow \quad h_2 = \dfrac{1100}{25 \times 11}$

$= 4$ m

(*b*) Mensuration [volume of solid bodies]

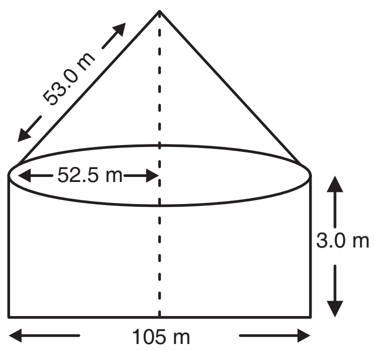
(*c*) $\because$ The height of spreadover mud is less in option-(*i*).

$\therefore$ The cost of digging the well is less in option-(*i*).

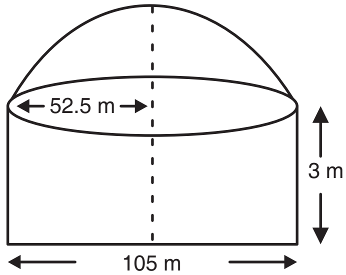
(*d*) Community-service and honesty.

Q15. *A contractor is entrusted to erect a tent for flood-victims. He is allowed a fixed amount for this task. He has two options :*

(*i*) *to erect a tent which is cylindrical upto a height of 3 m and conical above it. The diameter of the base is 105 m and slant height of the conical part is 53 m.*



(*ii*) *to erect the tent as described in option-(i), only replacing the conical part as a hemispherical part.*



The contractor chooses the option-(ii) and decides to donate the extra (difference) canvas to be used in this case.

(*a*) *How much canvas is donated by the contractor?*

(*b*) *Which mathematical concept is used in the above problem?*

(*c*) *By choosing the option-(ii) to erect the tent, which value is depicted by the contractor?*

Sol. (*a*) Total canvas used in option-(*i*) = [Curved surface area of the cylindrical part] + [Curved surface area of the conical part]

$= [2\pi rh + \pi rl]$

$= [(2 \times \frac{22}{7} \times 52.5 \times 3) + (\frac{22}{7} \times 52.5 \times 53)]\ m^2$

$= \frac{22}{7} \times 52.5\ [(2\times3) + 53]\ m^2$

$= \frac{22}{7} \times 52.5 \times 59\ m^2$

$= 9733\ m^2$

Total canvas used in option-(*ii*)

= [Curved surface area of cylindrical part] + [Curved surface area of hemispherical part]

$= [2\pi rh + 2\pi r^2]$

$= [(2 \times \frac{22}{7} \times 52.5 \times 3) + (2 \times \frac{22}{7} \times 52.5 \times 52.5)]\ m^2$

$= 2 \times \frac{22}{7} \times 52.5\ [3 + 52.5]\ m^2$

$= \frac{2 \times 22 \times 52.5}{7}\ [55.5]\ m^2$

$= \frac{2 \times 22 \times 52.5 \times 55.5}{7}\ m^2$

$= 18315\ m^2$

Difference in areas of canvas required in the above two options.

$= 18315 - 9733\ m^2$

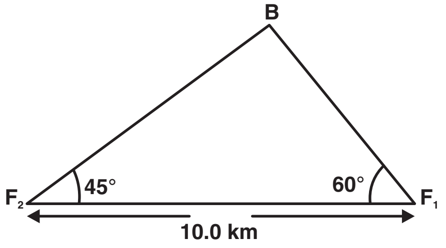
$= 8582\ m^2$

⇒ The contractor donated 8582 m^2 of canvas.

(*b*) Mensuration [volume of solid bodies]

(*c*) Patriotism

Q16. *A fire at a building B is reported on telephone to two fire stations F_1 and F_2, 10 km apart from each other on a straight road.*



F_1 observes that the fire is at an angle of 60° to the road and F_2 observes that it is at angle of 45° from it. The station F_1 sends its team.

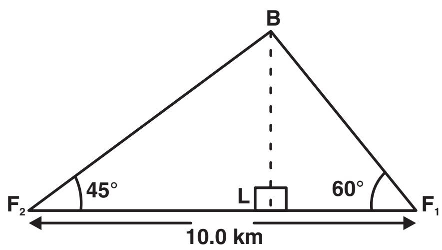
(*a*) *Why the team of station F_1 was sent?*

(*b*) *How much distance the station F_1 team will have to travel?*

(*c*) *Which mathematical concept is involved in the above problem?*

(*d*) *By sending its team, which value is depicted by the fire-station F_1?*

Sol. Let BL be the perpendicular from B on F_1F_2.



(*a*) $\because$ $BL \perp F_1F_2$

and $\angle LF_1B = 60°$;

$\angle LF_2B = 45°$

$\therefore$ From right-angled ΔF_1LB and ΔF_2LB, we have:

$$\sin 60° = \frac{BL}{F_1B} \text{ and } \sin 45° = \frac{BL}{F_2B}$$

$$\Rightarrow \quad F_1B = \frac{BL}{\sin 60°} \text{ and } F_2B = \frac{BL}{\sin 45°}$$

$\because$ $\sin\theta$ increases as θ increases from 0° to 90°.

$$\therefore \quad \sin 45° < \sin 60°$$

$$\Rightarrow \quad \frac{BL}{\sin 45°} > \frac{BL}{\sin 60°} \text{ or } F_2B > F_1B$$

$\therefore$ Distance of B from F_2 is more than that of from F_1. That is why the station F_1 sent its team to B.

(*b*) In right ΔF_1LB, $\cos 60° = \dfrac{F_1L}{F_1B}$

Let F_1B be x km

$$\Rightarrow \quad x \cos 60° = F_1L \quad \text{..........(1)}$$

$$\text{Also, } \frac{BL}{BF_1} = \sin 60°$$

$$\Rightarrow \quad BL = BF_1 \sin 60°$$

$$\Rightarrow \quad F_2L = x \sin 60° \quad \text{..........(2)}$$

In right ΔF_2LB, $\tan 45° = \dfrac{BL}{F_2L}$

$$\therefore \quad 1 = \frac{BL}{F_2L}$$

$$\Rightarrow \quad BL = F_2L \quad \text{..........(3)}$$

From (2) and (3), we get

$$F_1L + F_2L = 10$$

$$x \cos 60° + x \sin 60° = 10$$

$$\Rightarrow \quad x [\cos 60° + \sin 60°] = 10$$

$\because \cos 60° = \frac{1}{2}$, $\sin 60° = \frac{\sqrt{3}}{2}$

$$\Rightarrow \quad x \left[\frac{1}{2} + \frac{\sqrt{3}}{2}\right] = 10$$

$$\Rightarrow \quad x \left[\frac{1+\sqrt{3}}{2}\right] = 10$$

$$\Rightarrow \quad x \left[\frac{1+1.732}{2}\right] = 10$$

$$\Rightarrow \quad x \times 2.732 = 20$$

$$\Rightarrow \quad x = \frac{20}{2.732}$$

$$\Rightarrow \quad x = 7.32 \text{ km}$$

$\therefore$ Station F_1 team travelled 7.32 km

(*c*) Trigonometry [Heights and Distances]

(*d*) Promptness.

Q17. *Roshan is a wholesaler of electric-bulbs. He sells a box of 600 electric bulbs which contains 12 defective bulbs to an electrician. He gives 12 extra bulbs to the electrician. In the process of replacing the defective bulbs by non-defective bulbs, the electrician takes out one bulb at random from the box.*

(*a*) *What is the probability that it is non-defective bulb?*

(*b*) *Which mathematical concept is used in the above problem?*

(*c*) *By giving 12 extra non-defective bulbs to the electrician, which value is depicted by Roshan?*

Sol. (*a*) Total number of bulbs in the box = 600

Number of defective bulbs = 12

Number of non-defective bulbs = 600 – 12

= 588

Number of possible events = 600

Number of favourable events = 588

$$\therefore \quad \text{Probability of non-defective bulbs} = \frac{[\text{Number of favourable events}]}{[\text{Number of possible events}]}$$

$$= \frac{588}{600} = \frac{49}{50}$$

(*b*) Probability

(*c*) Honesty

Q18. *There are 40 students in class X, of whom 25 are girls and 15 are boys. They plan to go to help the earthquake victims in a camp. Their class-teacher has to select one student at random for the class representative.*

(*a*) *What is the probability that the student selected for class-representative is*

(*i*) *a girl* (*ii*) *a boy*

(*b*) *Which mathematical concept is used in the above problem?*

(*c*) *By planning to go for the help of earthquake victims, which value is depicted by the students of class X?*

Sol. (*a*) Total number of students in the class = 40

Number of boys = 15

Number of girls = 25

(*i*) Since, the class representative selected at random is to be a girl

∴ Number of possible outcomes = 40

Number of favourable outcomes = 25 [∵ There are 25 girls in the class]

∴ Probability of a girl to be the representative

$$= \frac{\text{Favourable outcomes}}{\text{Possible outcomes}}$$

$$= \frac{25}{40} \text{ or } \frac{5}{8}$$

(*ii*) Since, the class-representative selected at random is to be a boy

∴ Number of favourable outcomes = 15

⇒ Probability of a boy to be the representative $= \frac{15}{40}$ or $\frac{3}{8}$

(*b*) Probability

(*c*) Helping the persons, in need.

Q19. *In a school, students decided to plant trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be double of the class in which they are studying. If there are 1 to 12 classes in the school and each class has two sections, find how many trees were planted by the students. Which value is shown in this question?* [AI. CBSE 2014]

Sol. ∴ There are 12 classes in all.

∵ Each class has 2 sections.

∴ Number of plants planted by class I $= 1 \times 2 = 2$

Number of plants planted by class II $= 2 \times 2 = 4$

Number of plants planted by class III $= 3 \times 2 = 6$

Number of plants planted by class IV $= 4 \times 2 = 8$

....

....

Number of plants planted by class XII $= 12 \times 2 = 24$

The numbers 2, 4, 6, 8, ... 24 forms an A.P.

Here, $a = 2$, $d = 4 - 2 = 2$

∵ Number of classes = 12

∴ Number of terms (n) = 12

Now, the sum of n terms of the above A.P., is given by $S_n = \frac{n}{2}[2a+(n-1)d]$

$\therefore \quad S_{12} = \frac{12}{2}[2(2)+(12-1)2]$

$= 6\ [4 + (11 \times 2)]$

$= 6 \times 26 = 156$

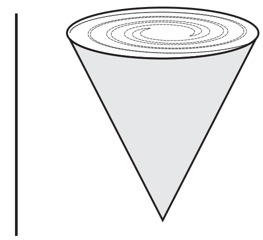
Thus, the total number of trees planted = 156

Value shown: To enrich polution free environment.

Q20. *Sushant has a vessel, of the form of an inverted cone, open at the top, of height 11 cm and radius of top as 2.5 cm and is full of water. Metallic spherical balls each of diameter 0.5 cm are put in the vessel due to which $\frac{2}{5}$ th of the water in the vessel flows out. Find how many balls were put in the vessel. Sushant made the arrangement so that the water that flows out irrigates the flower beds. What value has been shown by sushant?* [CBSE Delhi 2014]

Sol. Base radius of the cone (r) = 2.5cm

Height of the concial part (h) = 11cm



Using $V = \frac{1}{3}\pi r^2 h$, the volume of the conical vessel.

$$= \frac{1}{3}\times\frac{22}{7}\times\frac{25}{10}\times\frac{25}{10}\times 11\,\text{cm}^3 = \frac{11\times5\times25\times11}{3\times7\times10}\ \text{cm}^3$$

$$\frac{2}{5} \text{ of } \left[\frac{11\times5\times25\times11}{3\times7\times10}\right] = \frac{11\times5\times11}{3\times7}\ \text{cm}^3$$

$\therefore \quad \left(\frac{11\times5\times11}{3\times7}\right)\text{cm}^3$ of water is flown out due to 'n' spherical balls each of radius = 0.5 cm

$$\because \quad \text{Volume of } n \text{ balls} = \left[\frac{11\times5\times11}{3\times7}\right]\text{cm}^3$$

$$\Rightarrow \quad n[\textit{volume of one ball}] = \left[\frac{11\times5\times11}{3\times7}\right]\text{cm}^3$$

$$\Rightarrow \quad n\times\left[\frac{4}{3}\times\frac{22}{7}\times\frac{5}{10}\times\frac{5}{10}\times\frac{5}{10}\right] = \frac{11\times5\times11}{3\times7}$$

$$n = \frac{11\times5\times11}{3\times7}\times\frac{3}{4}\times\frac{7}{22}\times\frac{10}{5}\times\frac{10}{5}\times\frac{10}{5}$$

$$n = 11\times5 = 55$$

Thus, the required number of leadshots = 55

Value : To keep the plants green for pollution free surroundings.

Q21. *The angle of elevation of the top of a chimney from the foot of a tower is 60° and the angle of depression of the foot of the chimney from the top of the tower is 30°. If the height of the tower is 40m, find the height of the chimney. According to pollution control norms, the minimum height of a smoke emitting chimney should be 100m. State if the height of the above mentioned chimney meets the pollution norms. What value is discussed in this question?* [AI. CBSE (Foreign) 2014]

Sol. In the figure, the height of the tower (AB) = 40 m

Let the height of the chimney (CD) = h metre.

Now, in *rt.* ΔABC, we have :

Let BC = x

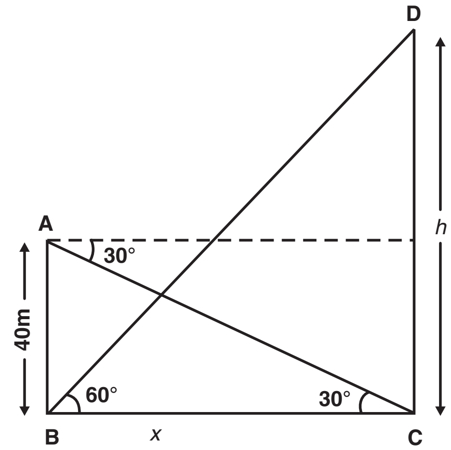
$$\therefore \quad \frac{40}{x} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad x = 40.\sqrt{3} \quad ...(1)$$

In *rt.* Δ BCD,

$$\frac{h}{x} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \quad x = \frac{h}{\sqrt{3}} \quad ...(2)$$



From (1) and (2) we have,

$$\frac{h}{\sqrt{3}} = 40\sqrt{3} \Rightarrow h = 40 \times \sqrt{3} \times \sqrt{3}$$

$$= 40 \times 3 \text{ m} = 120 \text{ m}$$

Thus, the height of the chimney is 120 m.

As, the minimum height of a chimney (according to the pollution control norms) should be 100 m

$\therefore$ 120 m > 100 m

$\therefore$ Thus, the above mentioned chimney meets the pollution norms.

Value : To keep pollution free environment.